Development of a 6 DoF Nonlinear Helicopter Model for the MPI Cybermotion Simulator

CANDIDATO:
Carlo Andrea Gerboni

RELATORI:
prof. ing. Lorenzo Pollini
prof. ing. Mario Innocenti
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Abstract

As part of the investigation of the flying vehicles of the future, particular attention is paid to the dynamics of helicopters. The state of the art of commercial helicopter simulators is composed by products that do not provide the implemented model used to realize the simulator itself and dynamics is unknown. Few complex models are developed to be used in simulators with motion due to the difficulties of obtaining reliable parameters to correct modeling the different parts of the helicopter.

In this work a fully nonlinear helicopter model is developed with the aim to be used in simulators with and without motion. This will allow to record data for identification purposes without use a real helicopter and to test different controls and commands to make to fly as easy as drive a car. All the different equations that describe the dynamics are studied and an accurate tuning work is done for all the unknown parameters.

To validate the model the handling qualities of the helicopter are analyzed following the standards defined by the Aeronautical Design Standard 33 (ADS33). Subsequently the model is tested by a helicopter pilot in a simulator without motion. The pilot performs different maneuvers evaluating them with a pilot rating scale made for simulators without motion.

Finally, preliminary test are done to understand the compatibility of the model in the Cybermotion Simulator situated in the Max Planck Institute of Tuebingen where this work was done. Test of the model in the Cybermotion simulator will be done after the publication of this work.
Nell’ambito dell’investigazione dei veicoli volanti del futuro particolare attenzione viene posta nella dinamica degli elicotteri. Lo stato dell’arte dei simulatori di volo di elicotteri commerciali si compone di prodotti che non forniscono il modello implementato per la realizzazione del simulatore stesso e la dinamica non é nota. Pochi modelli complessi sono stati sviluppati per permetterne un loro utilizzo in simulatori di moto a causa della difficoltà di reperire tutta le grandezze necessarie per modellare correttamente le varie componenti dell’elicottero. In questo lavoro di tesi un modello completamente non lineare di elicottero viene sviluppato con l’obiettivo di poter essere utilizzato su simulatori con e senza moto. Questo permetterá di registrare dati di volo utilizzabile per tecniche di identificazione senza l’uso di un vero elicottero e di testare sistemi di controllo e nuovi comandi che rendano volare semplice come guidare una macchina. Le differenti equazioni che compongono la dinamica delle varie parti vengono attenzionate e un accurato lavoro di tuning viene fatto su tutti i parametri incogniti. Per la validazione del modello sono analizzate le handling qualities del sistema seguendo gli standard definiti dall’Aeronautical Design Standard 33 (ADS33). Il modello successivamente viene testato con un pilota su un simulatore senza moto. Il pilota per usera differenti manovre valutandole sulla base di una scala di valutazione apposita per simulatori senza moto. Infine test preliminari vengono fatti per capire l’effettiva utilizzabilità del modello nel simulatore di moto presente al Max planck Institute di Tubinga. Le prove del modello nel Cyber-motion simulator verranno eseguite successivamente alla pubblicazione di questo lavoro.
Table of Contents

Abstract i
Riassunto iii
Notation xi

1 Aim of the work and state of the art 1
1-1 Aim of the work .......................................................... 1
1-2 State of the art ............................................................ 2
1-3 Overview of the thesis .................................................... 4

2 Mathematical model 5
2-1 Introduction ............................................................... 5
2-2 Helicopter forces and moments ....................................... 5
2-3 Main rotor ................................................................. 7
  2-3-1 Main rotor moments ................................................ 12
  2-3-2 Rotor torque ......................................................... 13
  2-3-3 Rotor inflow calculation .......................................... 14
  2-3-4 Ground effect on inflow .......................................... 15
  2-3-5 Summary of the main rotor equations ......................... 16
2-4 Tail rotor ................................................................. 17
  2-4-1 Summary of the tail rotor equations ......................... 18
2-5 Fuselage ................................................................. 19
  2-5-1 Summary of the fuselage equations ......................... 20
2-6 Empennage ............................................................... 21
  2-6-1 Summary of the empennage equations ....................... 23
2-7 Flight control system .................................................. 24
2-8 Scheme of the model ................................................... 27
Table of Contents

3 First validation of the dynamic behavior .................................................. 29
  3-1 Introduction .................................................................................. 29
  3-2 PID controllers ................................................................. 29
    3-2-1 Attitude control .......................................................... 31
    3-2-2 Rate-gyro ....................................................................... 32
    3-2-3 Altitude control .......................................................... 32
    3-2-4 Hover control ............................................................. 32
  3-3 Unity ................................................................. 34
    3-3-1 Ground effect in visualization ........................................... 35
    3-3-2 Force and moments test .................................................. 37

4 Validation with the ADS-33 ........................................................ 41
  4-1 Introduction ........................................................................... 41
  4-2 Aeronautical Design Standard ....................................................... 41
  4-3 Attitude quickness test ............................................................. 42
  4-4 Frequency response ............................................................... 44

5 Test with a pilot in a simulator without motion ....................................... 53
  5-1 Introduction ........................................................................... 53
  5-2 Pilot rating scale ................................................................. 53
  5-3 Fixed base simulator and the environment ..................................... 55
  5-4 Maneuvers evaluated with the PRS IAI ........................................ 57
  5-5 ADS-33 vertical remask ............................................................ 59

6 Future steps ...................................................................................... 63
  6-1 MPI Cybermotion Simulator ......................................................... 63

A Appendix A ......................................................................................... 67
  A-1 Reference systems ................................................................... 67
  A-2 Angular coordinates of the aircraft ............................................ 69
  A-3 Components of gravitational acceleration along the aircraft axes ........................................................................ 70
  A-4 The rotor system ...................................................................... 70

B Helicopter Parameters .......................................................................... 73
List of Figures

1-1 MPI Cybermotion simulator .................................................. 2
1-2 Yamaha R-Max ................................................................. 3
1-3 Comparison between rotors: (a) Sikosky UH-60; (b) Yamaha R-Max .......... 4

2-1 Helicopter components ...................................................... 6
2-2 Body axes reference system .................................................. 6
2-3 Forces and moments acting on a rotor hub .............................. 8
2-4 Azimuth angle ................................................................. 10
2-5 Swashplate tilt ................................................................. 11
2-6 Ground effect on a helicopter in hovering flight ....................... 15
2-7 Tail rotor subsystem .......................................................... 17
2-8 Variation of fuselage aerodynamic force coefficients with incidence angles 20
2-9 Horizontal and vertical stabilizer ......................................... 22
2-10 Influence of rotor downwash on tail surfaces ......................... 23
2-11 Flight control system ........................................................ 25
2-12 Simulation model .............................................................. 27

3-1 Closed loop system ............................................................ 30
3-2 PID actions ................................................................. 30
3-3 Roll control ................................................................. 32
3-4 Pitch control ................................................................. 32
3-5 Yaw control ................................................................. 32
3-6 Heave control ............................................................... 33
3-7 Linear velocities controlled by PID ...................................... 34
3-8 Rotational velocities controlled by PID ............................... 34
3-9 Euler angles controlled by PID .......................................... 35
3-10 A simple environment made with Unity ............................. 36
List of Figures

3-11 Ground reaction scheme along z ............................................. 36
3-12 Simulink scheme of ground reactions ........................................ 38
3-13 Unity desktop with the arrows added to the helicopter ................. 39
3-14 Test with arrows ............................................................... 39
4-1 Response to pulse lateral cyclic input ........................................ 42
4-2 Definition of the minimum $\Delta \phi$ .......................................... 43
4-3 Illustration of lateral cyclic inputs for roll attitude quickness: (a) input with return to the original tri position; (b) input maintaining the new attitude to have good compliance of the results ........................................ 44
4-4 Lateral cyclic inputs: (a) from real flight data; (b) made with MATLAB/Simulink ...................................................... 45
4-5 Roll quickness model response .................................................. 46
4-6 Roll quickness real helicopter response ....................................... 47
4-7 Pitch quickness model response .................................................. 48
4-8 Pitch quickness real helicopter response ....................................... 48
4-9 Definition of the parameters obtained from the frequency response .... 49
4-10 ADS-33 requirements for the frequency responses ......................... 49
4-11 Sweeping sinusoid used for the identification ................................ 50
5-1 Pilot Rating Scale of IAI .......................................................... 54
5-2 Panolab room ........................................................................... 55
5-3 Suggested course for the vertical remsk ........................................ 56
5-4 Suggested course for acceleration-deceleration maneuver .............. 56
5-5 Unity environment ..................................................................... 57
5-6 Wittenstein platform .................................................................. 58
5-7 ADS-33 vertical remsk maneuver description ................................ 60
6-1 SIMONA simulator in Delft ......................................................... 64
6-2 HELIFLIGHT simulator in Liverpool ............................................ 64
6-3 MPI Cybermotion Simulator .......................................................... 65
A-1 Reference axes frame ................................................................ 67
A-2 Body axes frame ....................................................................... 68
A-3 The fuselage Euler angles: (a) yaw; (b) pitch; (c) roll .................... 70
A-4 The hub and blade reference axes systems .................................... 71
B-1 UH-60 dimensions .................................................................... 73
List of Tables

3-1  PID parameters ........................................................................... 33
B-1  Aircraft mass and inertia ................................................................. 73
B-2  Center of gravity position ............................................................... 74
B-3  Main rotor .................................................................................... 74
B-4  Empennage .................................................................................. 75
B-5  Tail rotor ..................................................................................... 75
Notation

\( a_0 \) main rotor blade lift curve slope (1/rad)
\( a_0T \) tail rotor blade lift curve slope (1/rad)
\( c \) rotor blade chord (m)
\( g \) acceleration due to gravity (m/s\(^2\))
\( g_{1c0}, g_{1c1} \) lateral cyclic stick-blade angle gearing constants
\( g_{1s0}, g_{1s1} \) longitudinal cyclic stick-blade angle gearing constants
\( g_{cc0}, g_{cc1} \) collective lever-lateral cyclic blade angle gearing constants
\( g_{cT0} \) pedal/collective lever-tail rotor control run gearing constant
\( h_{fn} \) height of fin centre of pressure above centre of mass along negative z-axis (m)
\( h_R \) height of main rotor hub above above centre of mass (m)
\( h_T \) height of tail rotor hub above above centre of mass (m)
\( k_{\phi}, k_p \) feedback gains in roll axis control system (rad/rad, rad/(rad/s))
\( k_{\lambda_f} \) main rotor downwash factor at fuselage
\( k_{\lambda_{fn}} \) main rotor downwash factor at fin
\( k_{\lambda_T} \) main rotor downwash factor at tail rotor
\( k_{\lambdaTp} \) main rotor downwash factor at tail plane
\( k_{q}, k_{\dot{q}} \) feedback gains in pitch axis control system (rad/rad, rad/(rad/s))
\( l_f \) fuselage reference length (m)
\( l_{fn} \) distance of fin centre of pressure aft of centre of mass along negative x-axis (m)
\( l_T \) distance of tail rotor hub aft of centre of mass (m)
\( l_{tp} \) distance of tail plane centre of pressure aft of centre of mass (m)
\( p, q, r \) angular velocities components of helicopter about fuselage x-, y-, z-axis (rad/s)
\( p_{pk}/\Delta\phi \) attitude quickness parameter (1/sec)
\( s \) rotor solidity = \( N_b c / \pi R \)
\( s_T \) tail rotor solidity
\( u, v, w \) translational velocities components of helicopter along fuselage x-, y-, z-axis (m/s)
\( v_i \) induced velocity at disc (m/s)
\( w \) velocity along aircraft z-axis (ms)
\( w_\lambda \) \( w - k_{\lambda_f} \Omega R \lambda_0 \) total downwash over fuselage (m/s)
$x_c g$  
\begin{center}
\text{centre of gravity (centre of mass) location forward of fuselage reference point (m)}
\end{center}

$z_g$  
\begin{center}
\text{distance of ground below rotor (m)}
\end{center}

$C_Q$  
\begin{center}
\text{main rotor torque coefficient}
\end{center}

$C_T$  
\begin{center}
\text{rotor thrust coefficient}
\end{center}

$C_{T_T}$  
\begin{center}
\text{tail rotor thrust coefficient}
\end{center}

$C_{yf_n}$  
\begin{center}
\text{normalized sideforce on fin}
\end{center}

$C_{ztp}$  
\begin{center}
\text{normalized tailplane force}
\end{center}

$I_{\beta}$  
\begin{center}
\text{flap moment of inertia (kgm$^2$)}
\end{center}

$I_R$  
\begin{center}
\text{moment of inertia of rotor (kgm$^2$)}
\end{center}

$I_{xx}, I_{yy}, I_{zz}$  
\begin{center}
\text{moments of inertia of helicopter about the x-,y-,z-axis (kgm$^2$)}
\end{center}

$I_{xz}$  
\begin{center}
\text{product of inertia of helicopter about the x-,z-axis (kgm$^2$)}
\end{center}

$K_{\beta}$  
\begin{center}
\text{centre-spring rotor stiffness (N m/rad)}
\end{center}

$L, M, N$  
\begin{center}
\text{external aerodynamic moments about the x-,y-,z-axis (N m)}
\end{center}

$L_{f}, M_{f}, N_{f}$  
\begin{center}
\text{fuselage aerodynamic moments about centre of gravity (N m)}
\end{center}

$L_{fn}, N_{fn}$  
\begin{center}
\text{fin aerodynamic moments about centre of gravity (N m)}
\end{center}

$L_T, M_T, N_T$  
\begin{center}
\text{tail rotor moments about centre of gravity (N m)}
\end{center}

$M_a$  
\begin{center}
\text{mass of helicopter (kg)}
\end{center}

$M_{h}, L_{h}$  
\begin{center}
\text{main rotor hub pitch and roll moments (N m)}
\end{center}

$M_{R}, L_{R}$  
\begin{center}
\text{main rotor pitch and roll moments (N m)}
\end{center}

$M_{tp}$  
\begin{center}
\text{tail plane pitching moment (N m)}
\end{center}

$N_b$  
\begin{center}
\text{number of blades on main rotor}
\end{center}

$N_H$  
\begin{center}
\text{yawing moment due to rotor about rotor hub (N m)}
\end{center}

$Q_R$  
\begin{center}
\text{main rotor torque (N m)}
\end{center}

$R$  
\begin{center}
\text{rotor radius (m)}
\end{center}

$R_T$  
\begin{center}
\text{tail rotor radius}
\end{center}

$S_{\beta}$  
\begin{center}
\text{Stiffness number}
\end{center}

$S_{fn}$  
\begin{center}
\text{fin area (m$^2$)}
\end{center}

$S_{p}, S_{s}$  
\begin{center}
\text{fuselage plan and side area (m$^2$)}
\end{center}

$S_{tp}$  
\begin{center}
\text{tail plane area (m$^2$)}
\end{center}

$T$  
\begin{center}
\text{main rotor thrust (N)}
\end{center}

$T_{ige}$  
\begin{center}
\text{rotor thrust in-ground effect (N)}
\end{center}

$T_{oge}$  
\begin{center}
\text{rotor thrust out-of-ground effect (N)}
\end{center}

$T_T$  
\begin{center}
\text{tail rotor thrust (N)}
\end{center}

$V_{fn}$  
\begin{center}
\text{total velocity incident on fin (m/s)}
\end{center}

$V_{tp}$  
\begin{center}
\text{total velocity incident on tail plane (m/s)}
\end{center}

$X, Y, Z$  
\begin{center}
\text{external aerodynamic forces acting along the x-,y-,z-axis (N)}
\end{center}

$X_f, Y_f, Z_f$  
\begin{center}
\text{components of X, Y, Z from fuselage (N)}
\end{center}

$X_{hw}, Y_{hw}$  
\begin{center}
\text{rotor forces in hub/wind axis system (N)}
\end{center}

$X_R, X_T$  
\begin{center}
\text{components of X from main and tail rotors (N)}
\end{center}

$X_{tp}, X_{fn}$  
\begin{center}
\text{components of X from empennage (N)}
\end{center}

$Y_{fn}$  
\begin{center}
\text{aerodynamic sideforce acting on fin (N)}
\end{center}

$Y_T$  
\begin{center}
\text{component of Y force from tail rotor (N)}
\end{center}

$Z_{tp}$  
\begin{center}
\text{component of Z force from tail plane (N)}
\end{center}

$\alpha_{1cw}$  
\begin{center}
\text{effective cosine component of one-per-rev rotor blade incidence (rad)}
\end{center}

$\alpha_{1sw}$  
\begin{center}
\text{effective sine component of one-per-rev rotor blade incidence (rad)}
\end{center}

$\alpha_{tp}$  
\begin{center}
\text{incidence of resultant velocity to tail plane}
\end{center}
<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\alpha_{tp})</td>
<td>zero-lift incidence angle on tail plane (rad)</td>
</tr>
<tr>
<td>(\beta_f)</td>
<td>sideslip angle at fuselage (rad)</td>
</tr>
<tr>
<td>(\beta_{fn})</td>
<td>sideslip angle at fin (rad)</td>
</tr>
<tr>
<td>(\beta_0, \beta_{1c}, \beta_{1s})</td>
<td>rotor blade coning, longitudinal and lateral flapping angles (subscript (w) denotes hub/wind axes) - in multi-blade coordinates (rad)</td>
</tr>
<tr>
<td>(\beta_{fn0})</td>
<td>zero-lift sideslip angle on fin (rad)</td>
</tr>
<tr>
<td>(\delta_0)</td>
<td>main rotor profile drag coefficient</td>
</tr>
<tr>
<td>(\delta_2)</td>
<td>main rotor lift dependent profile drag coefficient</td>
</tr>
<tr>
<td>(\gamma)</td>
<td>Lock number</td>
</tr>
<tr>
<td>(\gamma_s)</td>
<td>shaft angle (positive forward, rad)</td>
</tr>
<tr>
<td>(\eta_{1s}, \eta_{1c})</td>
<td>pilot’s collective lever and cyclic stick position (positive up, aft and to port)</td>
</tr>
<tr>
<td>(\eta_{0s}, \eta_{1c})</td>
<td>cyclic gearing constants</td>
</tr>
<tr>
<td>(\lambda_0, \lambda_{1c}, \lambda_{1s})</td>
<td>rotor uniform and first harmonic inflow velocities in hub axes (normalized by (\Omega\ R))</td>
</tr>
<tr>
<td>(\lambda_{0T})</td>
<td>tail rotor uniform inflow component</td>
</tr>
<tr>
<td>(\lambda_\beta)</td>
<td>flap frequency ratio</td>
</tr>
<tr>
<td>(\chi)</td>
<td>main rotor wake angle (rad)</td>
</tr>
<tr>
<td>(\chi_{1s}, \chi_{2})</td>
<td>wake angle limits for downwash on tail (rad)</td>
</tr>
<tr>
<td>(\lambda_{tp})</td>
<td>normalized downwash at tail plane</td>
</tr>
<tr>
<td>(\mu)</td>
<td>advance ratio (V/\Omega\ R)</td>
</tr>
<tr>
<td>(\mu_T)</td>
<td>normalized velocity at tail rotor</td>
</tr>
<tr>
<td>(\mu_{tp})</td>
<td>normalized velocity at tail plane</td>
</tr>
<tr>
<td>(\mu_z, \mu_y, \mu_x)</td>
<td>velocities of the rotor hub in hub/axis axes (normalized by (\Omega\ R))</td>
</tr>
<tr>
<td>(\mu_{zT})</td>
<td>total normalized tail rotor inflow velocity</td>
</tr>
<tr>
<td>(\theta_0)</td>
<td>collective pitch angle (rad)</td>
</tr>
<tr>
<td>(\theta, \phi, \psi)</td>
<td>Euler angles defining the orientation of the aircraft relative to the Earth (rad)</td>
</tr>
<tr>
<td>(\theta_{0T})</td>
<td>tail rotor collective pitch angle (rad)</td>
</tr>
<tr>
<td>(\theta_{0T}')</td>
<td>tail rotor collective pitch angle after delta 3 correction (rad)</td>
</tr>
<tr>
<td>(\theta_{1s}, \theta_{1c})</td>
<td>longitudinal and lateral cyclic pitch (subscript (w) denotes hub/wind axes) (rad)</td>
</tr>
<tr>
<td>(\theta_{tw})</td>
<td>main rotor blade linear twist (rad)</td>
</tr>
<tr>
<td>(\rho)</td>
<td>air density ((kg/m^3))</td>
</tr>
<tr>
<td>(\tau_p)</td>
<td>phase delay between attitude response and control input at high frequency (s)</td>
</tr>
<tr>
<td>(\omega_{bw})</td>
<td>bandwidth frequency for attitude response (rad/sec)</td>
</tr>
<tr>
<td>(\psi)</td>
<td>rotor blade azimuth angle, positive in direction of rotor rotation (rad)</td>
</tr>
<tr>
<td>(\psi_w)</td>
<td>rotor sideslip angle (rad)</td>
</tr>
<tr>
<td>(\Omega)</td>
<td>main rotor speed (rad/s)</td>
</tr>
<tr>
<td>(\Omega_T)</td>
<td>tail rotor speed (rad/s)</td>
</tr>
</tbody>
</table>
Chapter 1

Aim of the work and state of the art

1-1 Aim of the work

The population growth in last ten years in combination with the growth of the mechanical technology led to considering the prevailing congestion problems with ground-based transportation as one of the most important problem to solve. Today many families have more than one car and public transport are more everyday especially in countries with a large population growth.

The European myCopter project (http://www.mycopter.eu/) [1] aims to pave the way for the Personal Aerial Vehicles (PAVs) to be used by the general public within the context of such a transport system. The idea is that the PAVs can be used for travelling between homes and working places, and for flying at low altitude in urban environments. Such PAVs should be fully or partially autonomous without requiring ground-based air traffic control. Furthermore, they should operate outside controlled airspace while current air traffic remains unchanged, and should later be integrated into the next generation of controlled airspace.

The Max Planck Institute (MPI) for Biological Cybernetics is one of the partner of this project (http://www.kyb.tuebingen.mpg.de/). In details the studies conducted in this institute has the aim to make a PAV as easy to fly as it is to drive a car. In these studies rotorcraft vehicles are considered as the main reference since their dynamics and kinematics best reflect those of a PAV. In this contest it seems clear that a model that replicate the behavior of a helicopter is useful for several reasons. Moreover a key facility which contributes to the aims of these studies is the MPI’s Cybermotion Simulator (CMS) shown in Figure 6-3

So, the aim of this thesis is the development of a fully nonlinear helicopter model that can be used in a simulator with motion to replicate the experience of flight.

The helicopter model in combination with the CMS can be used for several aims related with the projects of the MPI. The most important are:

- Record flight data that can be used to test identification methods
- Training of pilots at performing specific maneuvers for system identification purposes
2 Aim of the work and state of the art

- Investigation of pilots behavior
- Implementation of new control systems to be tested in simulation with human in-the-loop
- Testing of new driving controls
- Training of non-expert pilots

1-2 State of the art

In terms of simulators there is a big difference between airplanes and helicopters. Many simulators exist based on the dynamics of airplanes, commercial as academic, while works on helicopter simulators are not so common. The reason why it happens is related with the difficulty of accurately implementing the components of the vehicle and of obtaining reliable aerodynamic parameters. For sure the most important problem is the presence of the rotors. Replicate the behavior of the rotorcraft taking in account all the different phenomenon that happen in the main rotor is quite impossible. Wakes, inflow, air streaming and many other factors that involve the air passing through the blades really change a lot in every flight conditions. Almost ever is necessary to have good approximation of the wind tunnel data to describe the interaction air-vehicle (but this is a problem of the airplanes too). Often the solution found to overcome this problem is to build a model that works only in defined conditions like for examples the hover, the takeoff, the forward flight, etc, without make the connection between this flight states.

This work borns due to the impossibility to have a helicopter model. Commercial simulators as academic ones are not good to achieve the aims listed above. Different reasons are behind this statement.
Commercial simulators don’t give the code of the model developed. No one information is known about the dynamics implemented, about which effects are described and which not, which simplifications or approximations were done. Changing something in the simulator is not possible. Moreover the helicopters that can be used in a commercial simulator are the ones chosen by the producers and so, if a comparison with exactly one helicopter is required and that helicopter is not inserted in the simulator, that comparison is not executable. Simulators, also used previously in the MPI, allow to replicate en entire experience of flight, from the takeoff to the landing, but no modifications can be done due to the absence of the source code. Other commercial simulators are instead downloadable with open source code (e.g. Flightgear) but these simulators use linearized model and so the transitions between the different flight conditions are not smooth. Moreover not all the effect that the pilot feels when he flies are reproduced.

From the point of view of the literature instead is in the absence of many works based on full size helicopters. A huge literature is based on the small size unmanned helicopters. The most common model described is the Yamaha R-Max shown in Figure 1-2.

Small helicopters are largely for several aims from test new control algorithms to development guidance algorithms, motion planning projects and so on. Unfortunately the dynamics of a small helicopter is simplified compared with a real size helicopter. Figure 1-3 shows how different are the rotors of the Sikosky UH-60 model used in this thesis and the one of the Yamaha.

The main difference is the presence of a stabilizer bar in the small helicopter that reduce the effects of the roll of the blades. Moreover the flapping equations are simplified because define the condition of quasi-state is easier. If the literature of the small size helicopters is huge is not possible to say the same for the real size helicopters. The largest literature is american
and especially NASA reports. However, American papers are directed towards study on same specific characteristics. Many works are based on the study of modeling the interactions between air and fuselage. Find the relations to describe the wind tunnel results in equations is largely investigated. Often, the description of the rotor is simplified and many interactions neglected because it is not the point of the work. These simplifications are not good when the aim of the work is to make a model that can be used in all the different flight conditions by a pilot. The impingement of the different components on the other can be reproduced only when every parameters is exact.

All that is written till now explains the reasons why this work was developed. Now a description of the different parts of the thesis is shown.

1-3 Overview of the thesis

This thesis shows all the steps necessary to guide a model that can be used in a flight simulator. First of all, in next chapter the model is described. All the different subsystems are detailed with particular attention on the main problems found and on the unknown parameters. In chapter 3 a description of the experimental setup that is used for all the test to validate the model is done. The controllers implemented to do some test are shown as well as the environment. In chapter 4 the model is validate using an international standard for military rotorcraft while in chapter 5 are listed all the maneuver performed by a pilot with his evaluations. Finally in chapter 6 are shown the future steps plus a description of the MPI Cybermotion Simulator.
Chapter 2

Mathematical model

2-1 Introduction

The helicopter can be viewed as a complex arrangement of interacting subsystems. The most difficult to describe is without doubt the main rotor. The rotor blades bend and twist under the influence of unsteady and nonlinear aerodynamic loads, which are themselves a function of the blade motion. The calculation of aerodynamics loads is viewed as the blade motion relative to the air and hence the motion of the hub as well as the motion of the blades relative to the hub. Expressions for the accelerations of the fuselage center of gravity and a rotor blade element are derived in Appendix A. In this work the helicopter has a six degree of freedom (DoF) flight mechanics formulation for the fuselage, with the quasi-steady rotor taking up its new position relative to the fuselage instantaneously. Sections following show expressions for the forces and moments acting on the various components of the helicopter and how the combined forces and moments on these elements are assembled with the inertial and gravitational to form the overall helicopter equations of motion.

2-2 Helicopter forces and moments

The helicopter is consists of five different parts as shown in Figure 2-1. These parts are the fuselage that is the body of the vehicle, the main rotor that produces the thrust to lift, the tail rotor that allows to turn on the right and on the left, the empennage that helps to stabilize the motion and the powerplant that changes the rotational speed of the rotor.

The forces and moments are referred to a system of body-fixed axes centered at the aircraft’s center of mass Figure 2-2.

The axes will be oriented at an angle relative to the principal axes of inertia, with the x direction pointing forward along some convenient fuselage reference line. The equations of motion for the six fuselage DoFs are assembled by applying Newton’s laws of motion relating the applied forces and moments to the resulting translational and rotational accelerations.
Figure 2-1: Helicopter components

Figure 2-2: Body axes reference system
2-3 Main rotor

Forces

\[
\dot{u} = -(wq - vr) - g\sin(\theta) + \frac{X}{Ma} \tag{2-1}
\]

\[
\dot{v} = -(ur - wp) + g\cos(\theta)\sin(\phi) + \frac{Y}{Ma} \tag{2-2}
\]

\[
\dot{w} = -(vp - uq) + g\cos(\theta)\cos(\phi) + \frac{Z}{Ma} \tag{2-3}
\]

Moments

\[
I_{xx}\dot{p} = (I_{yy} - I_{zz})qr + I_{xz}(\dot{r} + pq) + L \tag{2-4}
\]

\[
I_{yy}\dot{q} = (I_{zz} - I_{xx})rp + I_{xz}(r^2 - p^2) + M \tag{2-5}
\]

\[
I_{zz}\dot{r} = (I_{zz} - I_{yy})pq + I_{xz}(\dot{p} - qr) + N \tag{2-6}
\]

where \(u\), \(v\) and \(w\) and \(p\), \(q\) and \(r\) are the inertial velocities in the moving axes system; \(\phi\), \(\theta\) and \(\psi\) are the Euler rotations defining the orientation of the fuselage axes with respect to earth and hence the components of the gravitational force. \(I_{xx}\), \(I_{yy}\), etc., are the fuselage moments of inertia about the reference axes and \(M_a\) is the aircraft mass.

The different parts will be now discussed in details defining all the equations used to describe them mathematically. This treatment is mainly based on the theory expressed in [2]

2-3 Main rotor

The main rotor is without doubt the most difficult part to describe due to the the complex interaction between the blades and the air. With to respect to the body axes reference system described above, the main rotor hub is forward tilted of a forward shaft angle to compensate the rear position of the center of gravity. Moreover, as explained in appendix A, all the forces are calculated in the hub-wind reference system and so two rotation are necessary to have the equations in body axes. Forces and moments acting on a rotor hub are shown in Figure 2-3 and in the hub-wind frame can be written as

\[
X_{hw} = \sum_{i=1}^{Nb} \int_0^R \{- (f_z - ma_zb)_{i} \beta_i \cos \psi_i - (f_y - ma_yb)_{i} \sin \psi_i + ma_zb \cos \psi_i\} \, dr_b \tag{2-7}
\]

\[
Y_{hw} = \sum_{i=1}^{Nb} \int_0^R \{ (f_z - ma_zb)_{i} \beta_i \sin \psi_i - (f_y - ma_yb)_{i} \cos \psi_i + ma_zb \sin \psi_i\} \, dr_b \tag{2-8}
\]

\[
Z_{hw} = \sum_{i=1}^{Nb} \int_0^R (f_z - ma_zb + ma_zb\beta_i)_{i} \, dr_b \tag{2-9}
\]
where the two forces using the small angles approximation are

\[ f_z = -l \cos \phi - d \sin \phi \approx l - d\phi \]  \hspace{1cm} (2-10)

\[ f_y = d \cos \phi - l \sin \phi \approx d - l\phi \]  \hspace{1cm} (2-11)

with \( l \) and \( d \) that are lift and drag described as in [?]. Performing the integration analytically the forces can be written in coefficient form as

\[ C_{xw} = \frac{X_{hw}}{\rho(\Omega R)^2\pi R^2} \]  \hspace{1cm} (2-12)

\[ C_{yw} = \frac{Y_{hw}}{\rho(\Omega R)^2\pi R^2} \]  \hspace{1cm} (2-13)

\[ C_{zw} = \frac{Z_{hw}}{\rho(\Omega R)^2\pi R^2} \]  \hspace{1cm} (2-14)

where \( C_{xw}, C_{yw} \) and \( C_{zw} \) are the hub force coefficients described as

\[ C_{xw} = \frac{a_0 s}{2} \left\{ \left( \frac{F_0^{(1)}}{2} + \frac{F_2^{(1)}}{4} \right) \right\} \beta_{1cw} + \frac{F_1^{(1)}}{2} \beta_0 + \frac{F_2^{(1)}}{4} \beta_{1sw} + \frac{F_1^{(2)}}{2} \]  \hspace{1cm} (2-15)

\[ C_{yw} = \frac{a_0 s}{2} \left\{ \left( -\frac{F_0^{(1)}}{2} + \frac{F_2^{(1)}}{4} \right) \right\} \beta_{1sw} - \frac{F_1^{(1)}}{2} \beta_0 - \frac{F_2^{(1)}}{4} \beta_{1cw} + \frac{F_1^{(2)}}{2} \]  \hspace{1cm} (2-16)
\[ C_{zw} = -C_T = -\frac{a_{0s}}{2}F_0^{(1)} \]  

(2-17)

In equation (2.17) \( C_T \) is the normalized thrust coefficient developed by the main rotor

\[ C_T = \frac{T_{mr}}{\rho(\Omega R)^2 \pi R^2} \]

this thrust depends by the air through the disc of the rotor as will be described in details in next section. The harmonic coefficients of equations (2.15) and (2.16) are given by the expressions

\[ F_0^{(1)} = \theta_0 \left( \frac{1}{3} + \frac{\mu^2}{2} \right) + \frac{\mu}{2} \left( \theta_{1sw} + \bar{p}_{hw} \right) + \left( \frac{\mu_z - \lambda_0}{2} \right) + \frac{1}{4} (1 + \mu^2) \theta_{tw} \]  

(2-18)

\[ F_1^{(1)} = \left( \frac{\alpha_{1sw}}{3} + \mu \left( \theta_0 + \mu_z - \lambda_0 + \frac{2}{3} \theta_{tw} \right) \right) \]  

(2-19)

\[ F_1^{(1)} = \left( \frac{\alpha_{1cw}}{3} - \frac{\beta_0}{2} \right) \]  

(2-20)

\[ F_2^{(1)} = \frac{\mu}{2} \left( \frac{\alpha_{1cw}}{2} + \frac{\bar{p}_{hw} - \lambda_{1sw}}{2} \right) - \mu \beta_0 \]  

(2-21)

\[ F_2^{(1)} = -\frac{\mu}{2} \left( \frac{\alpha_{1sw}}{2} + \frac{\theta_{1sw} + \beta_{1cw}}{2} + \mu \left( \theta_0 + \theta_{tw} \right) \right) \]  

(2-22)

\[ F_1^{(2)} = \frac{\mu^2}{2} \beta_0 \beta_{1sw} + \left( \mu_z - \lambda_0 - \frac{\mu}{4} \beta_{1cw} \right) \left( \alpha_{1sw} - \theta_{1sw} \right) \]  

- \frac{\mu}{2} \beta_{1sw} (\alpha_{1cw} - \theta_{1cw}) \theta_0 \left( \frac{\alpha_{1sw} - \theta_{1sw}}{3} + \mu (\mu_z - \lambda_0) - \frac{\mu^2}{4} \beta_{1cw} \right)

+ \frac{\mu}{2} \left( \frac{\mu_z - \lambda_0}{2} + \mu \left( \frac{3}{8} \bar{p}_{hw} - \lambda_{1sw} \right) + \frac{\beta_{1cw}}{4} \right)

+ \frac{\mu}{4} \theta_{1cw} \left( \bar{q}_{hw} - \lambda_{1cw} - \beta_{1sw} - \mu \beta_0 \right) - \frac{\delta u}{\alpha_0} \]  

(2-23)

\[ F_1^{(2)} = \left( \alpha_{1cw} - \theta_{1cw} - 2 \mu \beta_0 \right) \left( \mu_z - \lambda_0 - \frac{3}{4} \mu \beta_{1cw} \right) - \frac{\mu}{4} \beta_{1sw} \left( \alpha_{1sw} - \theta_{1sw} \right) \]  

(2-24)
In equations (2.18)-(2.24) a lot of new terms appear and is necessary define them. Let’s start with those are called advance ratios

\[
\mu = \sqrt{\frac{u_h^2 + v_h^2}{(\Omega R)^2}} \tag{2-25}
\]

\[
\mu_z = \frac{w_h}{(\Omega R)^2} \tag{2-26}
\]

The velocities \(u_h, v_h\) and \(w_h\) are the hub velocities in the hub system, oriented relative to the aircraft x-axis by the relative airspeed or wind direction in the \(x-y\) plane. \(\theta\) and \(\beta\) are respectively the pitch angle and the flapping angle of the blades. The blade pitch of the blades is directly changed by the pilot with the commands (how it is done it will be described in a later section) and the flapping angles are related with them. Proper expressions of these angles are not simple to find but a good approximations is describe them with a first order Fourier analysis obtained the so-called quasi-steady form

\[
\theta = \theta_0 + \theta_{1c} \cos(\Psi) + \theta_{1s} \sin(\Psi) \tag{2-27}
\]

\[
\beta = \beta_0 + \beta_{1c} \cos(\Psi) + \beta_{1s} \sin(\Psi) \tag{2-28}
\]

where \(\Psi\) is the rotor blade azimuth angle shown in Figure 2-4.

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{azimuth_angle.png}
\caption{Azimuth angle}
\end{figure}

In equation (2.27) \(\theta_0\) is the collective pitch and \(\theta_{1c}\) and \(\theta_{1s}\) are the longitudinal and lateral cyclic pitch respectively. The cyclic flapping can be interpreted as a tilt of the rotor disc.
in the longitudinal (forward) $\beta_{1c}$ and lateral (port) $\beta_{1s}$ planes while $\beta_0$ is the conic angle. Figure 2-5 gives a clear physical interpretation of these angles.

Describe the variations of the flapping angles due to the variation of the pitch angles is not simple and different models were tested in this work ([2],[3],[4]). To choose the best one for this model a comparison of the values of $\beta$ in function of $\theta$ comparing with real data cited in [5]. These values will be shown in the next section. Flapping equations used are

\[ \beta_0 = \gamma \left[ \frac{\theta_0}{8} (1 + \mu^2) + \frac{\theta_{tw}}{10} (1 + \frac{5}{6} \mu^2) + \frac{\mu}{6} \theta_{1s} - \frac{\lambda_0}{6} \right] \]  
(2-29)

\[ \beta_{1s} = \theta_{1c} + \left( -\frac{\sqrt{3}}{2} \mu \beta_0 \right) \frac{1}{(1 + \frac{1}{2} \mu^2)} \]  
(2-30)

\[ \beta_{1c} = -\theta_{1s} + \frac{-\frac{8}{3} \left[ \theta_0 - \frac{3}{4} \lambda_0 + \frac{3}{4} \mu \theta_{1s} + \frac{3}{4} \theta_{tw} \right]}{(1 - \frac{1}{2} \mu^2)} \]  
(2-31)

In the above formulas as in the harmonic ones appears the term $\lambda_0$. This term is called rotor uniform inflow velocity and it will be discussed with more attention in the next section. As for $\theta$ and $\beta$ is possible to apply the first order Fourier analysis to the inflow $\lambda$ in such a way to obtain the first longitudinal and lateral harmonic. Being a velocity is quite impossible to find a general expression that can be used equally if the helicopter is in hover as in forward flight. A good approximation is to consider the inflow as a function of the angular speed that is to write

\[ \lambda_{1c} = k \bar{\dot{q}} \]  
(2-32)

\[ \lambda_{1s} = k \bar{\dot{p}} \]  
(2-33)

where $k$ is a constant usually between 0.1 and 0.3 and

\[ \bar{\dot{p}} = \frac{p}{\Omega} \quad \bar{\dot{q}} = \frac{q}{\Omega} \]  
(2-34)
Finally using all the quantities defined till now rotated in the hub-wind reference system is possible to define the effective blade incidence angles as

\[
\alpha_{1sw} = \bar{p}_{hw} - \lambda_{1sw} + \beta_{1cw} + \theta_{1sw}
\]

(2-35)

\[
\alpha_{1cw} = \bar{q}_{hw} - \lambda_{1cw} - \beta_{1sw} + \theta_{1cw}
\]

(2-36)

The last parameter to describe is the blade profile drag coefficient \( \delta \). It can be written in terms of a mean value plus a thrust-dependent term to account for blade incidence as suggested in [6]

\[
\delta = \delta_0 + \delta_2 C_T^2.
\]

(2-37)

Now that all quantities in equations (2.18)-(2.24) are described, some physical interpretation can be made. These expressions for the rotor forces highlight that in the non-rotating hub-wind axes system, a multitude of physical effects combine to produce the resultants. While the normal force, the rotor thrust, is given by a relatively simple equation, the in-plane forces are very complex indeed. The \( F_0^{(1)} \beta_{1cw} \) and \( F_1^{(1)} \beta_0 \) components are the first harmonics of the product of the lift and flapping in the direction of motion and represent the contribution to X and Y from blades in the fore and aft positions. The terms \( F_{1s} \) and \( F_{1c} \) represent the contributions to X and Y from the induced and profile drag acting on the advancing and retreating blades.

### 2-3-1 Main rotor moments

Being the center of gravity not coincident with the center of the hub, forces generated with equations written in last section produce also couples. However to calculate moments other effects of the rotor need to be taken in account. These effects are the stiffness effect and the rotor torque generated by the rotation of the blades. The first effect can be approximated with a linear function of the flapping angles that is

\[
L_h = -\frac{N_h}{2} K_\beta \beta_{1s}
\]

(2-38)

\[
M_h = -\frac{N_h}{2} K_\beta \beta_{1c}
\]

(2-39)

The hub stiffness, described with \( K_\beta \) can be written in terms of the flap frequency ratio i.e.,

\[
K_\beta = (\lambda_\beta^2 - 1) \Omega^2
\]

(2-40)

where \( \lambda_\beta \) is the flap frequency and \( \Omega \) is the rotation speed. The stiffness of rotors changes a lot depends on the mechanical construction of them. In particular the stiffness of a hingeless rotor can be three or four times greater than that of an articulated rotor and this means that the greater hub moment is obtained with a hingeness rotor. This aspect is much important.
because the UH-60 has a fully articulated rotor with non flapping spring ([7],[5]) and so the flapping frequency is 1 and $K_\beta$ is 0. Hence for this particular helicopter there is no moment developed by the stiffness. To describe the second effect is necessary to introduce the rotor torque and this will be done in the following section.

### 2-3-2 Rotor torque

In the rotor shaft a dominant component is the rotor torque that produces the yaw moment plus smaller components in pitch and roll as said before. Referring to Figure 2-3 the yaw moment in the hub-wind axes can be obtained as

$$N_{hw} = \sum_{i=1}^{N_b} \int_0^R r_b (f_y - m a_{y_b})_i \, dr_b$$

(2-41)

Applying the same simplifications of equation (2.7)-(2.9) and neglecting all the inertia terms except the accelerating torque caused by the rotor angular acceleration, equation (2.41) becomes

$$\frac{N_{hw}}{\frac{1}{2} \rho (\Omega R)^2 \pi R^3 s a_0} = \frac{2 C_Q}{a_0 s} + \frac{2}{\gamma} \left( \frac{I_R}{N_b N_\beta} \right) \bar{\Omega}'$$

(2-42)

where

$$\bar{\Omega}' = \frac{\dot{\Omega}}{\Omega^2}$$

(2-43)

In this work the powerplant is not modeled and so the rotational speed is assumed constant and doesn’t change. This means that equation (2.43) is null and the only contribution in $N_{hw}$ is the aerodynamic torque coefficient written as

$$\frac{2 C_Q}{a_0 s} = \left( \frac{2}{a_0 s} \right) \left( \frac{Q_R}{\rho (\Omega R)^2 \pi R^3} \right)$$

(2-44)

where $Q_R$ is namely rotor torque. A good approximation for the aerodynamic coefficient is

$$C_Q \approx \frac{a_0 s}{2} \left( -(\mu_z - \lambda_0) \left( \frac{2 C_T}{a_0 s} \right) + \mu \left( \frac{2 C_{xw}}{a_0 s} \right) + \frac{\delta}{4 a_0} (1 + 3 \mu^2) \right)$$

(2-45)

Equation (2.45) allows an approximation for the rotor torque and with this is possible to evaluate the small contribution of the yaw action in pitch and roll writing

$$L_{HQ} = -\frac{Q_R}{2} \beta_{1c}$$

(2-46)

$$M_{HQ} = -\frac{Q_R}{2} \beta_{1s}$$

(2-47)
2-3-3 Rotor inflow calculation

One of the most difficult part in the modeling a helicopter is to describe the inflow passing through the blades. In previous sections the total inflow $\lambda_0$ is introduced. This parameter can’t be treated as a constant and it is a function of the thrust, the forward and the vertical speed. At the same time the rotor thrust is a function of the induced velocity $v$ that is related with the inflow by the expression

$$\lambda_0 = \frac{v}{\Omega R} \quad (2-48)$$

so there is a problem of two variables depending one each other. To solve this loop several methods are shown in [8]. The one adopted in this work is the most common (used for example in [2] and [9]) that is the Newton-Raphson method. So a zero function of the induced velocity is defined as

$$g_0 = \lambda_0 - \frac{C_T}{2\Lambda^2} \quad (2-49)$$

where

$$\Lambda = \mu^2 + (\lambda_0 - \mu z)^2 \quad (2-50)$$

Its derivative is

$$h_j = -\left(\frac{g_0}{d g_0/d \lambda_0}\right)_{\lambda=\lambda_0} \quad (2-51)$$

i.e.,

$$h_j = -\frac{\Lambda(2\lambda_0, \Lambda^2 \frac{1}{2} - C_T)}{2\Lambda^2 + \Lambda \frac{a_{0x}}{4} - C_T (\mu z - \lambda_0)} \quad (2-52)$$

The goal of the method is to find a value of $\lambda_0$ such that the function $g_0$ goes to zero. In this way in each step the Newton’s iterative scheme gives

$$\lambda_{0,j+1} = \lambda_{0,j} + f_j h_j(\lambda_j) \quad (2-53)$$

and with the new value of the inflow the thrust coefficient is calculated using equations (2.17), (2.18)

$$C_T = \frac{a_{0x}}{2} \left( \theta_0 \left( \frac{1}{3} + \frac{\mu^2}{2} \right) + \frac{\mu}{2} \left( \theta_{1sw} + \frac{\bar{p}}{2} \right) + \left( \frac{\mu z - \lambda_0}{2} \right) \right) \frac{1}{4} (1 + \mu^2) \theta_{tw} \quad (2-54)$$

The Newton-Rhapson method provides rapid estimate of the inflow at time $t_{j+1}$ from a knowledge of conditions at time $t_j$. Because of in certain flight condition near the hover
the iteration can diverge a damping constant is added. The value chosen is 0.6 used by [2]. Therefore, with this iterative method is possible to find a solution for each flight condition without create a model divided in different flight scheme and so with no use of switch inside the code. This last point is fundamental to achieve a model that seems very close to a real helicopter. Moreover this method is simply to implement when the derivative can be written analytically as in this case.

2-3-4 Ground effect on inflow

Operating helicopters close to the ground introduces a range of special characteristics in the flight dynamic behavior. The most significant is the effect on the induced velocity at the rotor and hence the rotor thrust and power required. Different approach can be used to describe this effect. The method chosen for this model is the one described by Cheeseman and Bennett [10] and used in [2]. In this method the ground plane influence is modelled with a rotor of equal and opposite strength, in momentum terms, at an equidistance below the ground (Figure 2-6).

\[ \delta v_i = \frac{A_d v_i}{16\pi z_g^2} \]  

(2-55)

where \(z_g\) is the distance of the ground below the rotor disc and \(A_d\) is the rotor disc area. Now using the approach described in [11] the ratio of the induced velocity in and out of ground effect is
\[
\frac{v_i/IGE}{v_i/OGE} = \frac{1}{1 - \frac{R^2}{16z^2}}
\] (2-56)

does in non dimensional form becomes

\[
\frac{\lambda_0/IGE}{\lambda_0/OGE} = \left[ 1 - \frac{1}{16H^2} \right]^{\frac{3}{2}}
\] (2-57)

where \(H\) is the non dimensional rotor height. Equation (2.57) enables ground effect calculations to be made whilst in the hover. Although it is recognised that ground effect diminishes rapidly with forward speed it is necessary to obtain expressions predicting the effect of flying close to the ground at some forward airspeed. Cheeseman and Bennett derived a similar expression to that given by equation (2.57) using the same potential flow theory, but varying the strength of the source to account for the forward velocity. The resulting expression to account for ground effect in forward flight is given in non-dimensional form as,

\[
\frac{\lambda_0/IGE}{\lambda_0/OGE} = 1 - \frac{1}{16H^2(1 + (\frac{\mu}{\lambda_0/OGE})^2)^\frac{3}{2}}
\] (2-58)

2-3-5 Summary of the main rotor equations

In previous sections all the different actions acting on the main rotor are described. With the transformations described in Appendix A all these actions are converted from the hub-wind reference system to the body axes reference system. To not create confusion in this summary same notation used before will be used but with the add of a index “b” to show that those forces are now in body axes.

\[
X_{mr} = X_{hw}^b
\] (2-59)

\[
Y_{mr} = Y_{hw}^b
\] (2-60)

\[
Z_{mr} = Z_{hw}^b
\] (2-61)

\[
L_{mr} = L_h^b - h_RY_{mr}
\] (2-62)

\[
M_{mr} = M_h^b + h_RX_{mr} - x_hZ_{mr}
\] (2-63)

\[
N_{mr} = N_{hw}^b + x_hY_{mr}
\] (2-64)
2-4 Tail rotor

As largely described in section (2.3.2) the main rotor produces a torque that in combination with the couple generated by the lateral force gives a yaw moment expressed by equation (2.60). Due to direction of the rotation of the blades can’t be changed by the pilot, if there are no other component the helicopter starts to turn, always in the same direction. The tail rotor action allows to the pilot to stabilize the yaw and to turn producing a moment that is opposite to the one of the main rotor. The basic equations for tail rotor forces and moments are similar to those for the main rotor. In fact is possible to think to the tail rotor as a small main rotor. A high-fidelity tail rotor model will require a sophisticated formulation for the normal and in-plane components of local induced inflow applying the same theory used to describe the behavior of the main rotor. However in this model it is ignored the non-uniform effects described above and derive the tail rotor forces and moments from simple considerations. This means that the components along $x$ and $z$ are neglected due to the small influence that they have and only the force along $y$ is taken in account. Figure 2-7 shows the forces acting on the tail

\[ C_{T_T} = \frac{T_T}{\rho(\pi R_T^2)(\Omega_T R_T)^2} \]  \hspace{1cm} (2-65)

Using the blade theory the tail rotor thrust can be written also as

\[ C_{T_T} = \frac{a_0 T_T s_T}{2} \left( \frac{\theta_{0T}}{3} \left( 1 + \frac{3}{2} \mu_T^2 \right) + \frac{(\mu_{zT} - \lambda_{0T})}{2} \right) \]  \hspace{1cm} (2-66)

In figure $T_T$ represents the tail rotor thrust that normalized is recalled tail rotor thrust coefficient $C_{T_T}$.
and because of similar mechanical construction with the main rotor there is again a dependency problem with the inflow defined as

\[
\lambda_{0T} = \frac{C_{T_T}}{2(\mu_T^2 + (\mu_{zT} - \lambda_{0T})^2)^{\frac{1}{2}}}
\] (2-67)

that is solved exactly applying the Newton-Rhapson method as for the main rotor.

Looking to equation (2.62) new quantities need to be described. The tail rotor hub aerodynamic velocities are given by

\[
\mu_T = \frac{\sqrt{u^2 + (w - k_{\lambda T}\lambda_0 + q\lambda_T)^2}}{\Omega_T R_T}
\] (2-68)

\[
\mu_{zT} = \frac{-v + rT - pT}{\Omega_T R_T}
\] (2-69)

in which a factor \(k_{\lambda T}\) is included to take into account the influence of the main rotor inflow at the tail rotor. \(\theta^*_0T\) is the effective collective pitch

\[
\theta^*_0T = \frac{\theta_{0T} + k_3 \left(\frac{\gamma}{8\pi^2}\right)_T (\mu_{zT} - \lambda_{0T})}{1 - k_3 \left(\frac{\gamma}{8\pi^2}\right)_T (1 + \mu_T^2)}
\] (2-70)

in which \(\theta_{0T}\) is the pilot command given by the pedals.

### 2-4-1 Summary of the tail rotor equations

Neglecting the forces X and Z only three equations are implemented that are the Y force and its couples due to the horizontal and vertical distance from the center of mass

\[
Y_{TR} = \rho(\Omega_T R_T)^2(\pi R_T)^2 C_{T_T}
\] (2-71)

\[
L_{TR} = -h_T Y_{TR}
\] (2-72)

\[
N_{TR} = l_T Y_{TR}
\] (2-73)
The fuselage is the body of the vehicle and his mass changes a lot from model to model especially from military to civil helicopters. The flow around it is characterized by strong nonlinearities and the influence of the main rotor wake distorts it making its modeling really difficult to do. Most of the flight mechanics models are based on empirical fitting of wind tunnel data gathered at a limited range of dynamic pressure and fuselage angles of incidence. But in case in which tunnel data are not available like in this work in necessary to find approximated functions that can describe the interaction of the flow with the fuselage in each direction depending of the angle of incidence, laterally and frontally. Assuming that the fuselage is immersed in the uniform component (that means take in account the rotor downwash) is possible to write the angle of incidence and the velocity as

\[ \alpha_F = \tan^{-1}\left(\frac{w}{u}\right), \quad V_F = (u^2 + v^2 + w^2)^{\frac{1}{2}} \quad \lambda_0 < 0 \] (2-74)

\[ \alpha_F = \tan^{-1}\left(\frac{w_\lambda}{u}\right), \quad V_F = (u^2 + v^2 + w_\lambda^2)^{\frac{1}{2}} \quad \lambda_0 > 0 \] (2-75)

where

\[ w_\lambda = w - k_{\lambda f} \Omega R \lambda_0 \] (2-76)

and \( k_{\lambda f} \) is a constant taking into account the increase in downwash at the fuselage relative to the rotor disc. About this factor no information is available from different works studied ([7],[12]) and so an empirical solution was found trying different values till a value of 1 was chosen. For the lateral movement a sideslip angle is defined as

\[ \beta_F = \sin^{-1}\left(\frac{v}{V_F}\right) \] (2-77)

where \( S_p \) and \( S_s \) are the plan and side areas of the helicopter fuselage that means the middle area that is more or less always invested by the flow. Figure 2-8 shows the functions obtained with the different values of incidence angles while the table down shows the points used to obtained the functions just seen.

<table>
<thead>
<tr>
<th>( \alpha_f )</th>
<th>-180</th>
<th>-160</th>
<th>-90</th>
<th>-30</th>
<th>0</th>
<th>20</th>
<th>90</th>
<th>160</th>
<th>180</th>
</tr>
</thead>
<tbody>
<tr>
<td>( C_{xf} )</td>
<td>0.1</td>
<td>0.08</td>
<td>0</td>
<td>-0.07</td>
<td>-0.08</td>
<td>-0.07</td>
<td>0</td>
<td>0.08</td>
<td>0.1</td>
</tr>
<tr>
<td>( \alpha_f )</td>
<td>-180</td>
<td>-160</td>
<td>-120</td>
<td>-60</td>
<td>-20</td>
<td>0</td>
<td>20</td>
<td>60</td>
<td>120</td>
</tr>
<tr>
<td>( C_{zf} )</td>
<td>0</td>
<td>0.15</td>
<td>1.3</td>
<td>1.3</td>
<td>0.15</td>
<td>0</td>
<td>-0.15</td>
<td>-1.3</td>
<td>-1.3</td>
</tr>
<tr>
<td>( \alpha_f )</td>
<td>-205</td>
<td>-160</td>
<td>-130</td>
<td>-60</td>
<td>-25</td>
<td>25</td>
<td>60</td>
<td>130</td>
<td>155</td>
</tr>
<tr>
<td>( C_{mf} )</td>
<td>0.02</td>
<td>-0.03</td>
<td>0.1</td>
<td>0.1</td>
<td>-0.04</td>
<td>0.02</td>
<td>-0.1</td>
<td>-0.1</td>
<td>0.02</td>
</tr>
<tr>
<td>( \beta_f )</td>
<td>-90</td>
<td>-70</td>
<td>-25</td>
<td>0</td>
<td>25</td>
<td>70</td>
<td>90</td>
<td></td>
<td></td>
</tr>
<tr>
<td>( C_{nf} )</td>
<td>-0.1</td>
<td>-0.1</td>
<td>0.005</td>
<td>0</td>
<td>-0.005</td>
<td>0.1</td>
<td>0.1</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
2-5-1 Summary of the fuselage equations

Using coefficients found and choosing reasonable values for the Sp and Ss areas equations can be written as

\[ X_{fus} = \frac{1}{2} \rho S_p V_F^2 C_{xf} (\alpha_f) \]  \hspace{1cm} (2-78)

\[ Y_{fus} = \frac{1}{2} \rho S_s V_F^2 C_{xf} (\beta_f) \]  \hspace{1cm} (2-79)

\[ Z_{fus} = \frac{1}{2} \rho S_p l_f V_F^2 C_{zf} (\alpha_f) \]  \hspace{1cm} (2-80)

\[ L_{fus} = \frac{1}{2} \rho S_s l_f V_F^2 C_{nf} (\alpha_f) \]  \hspace{1cm} (2-81)

Figure 2-8: Variation of fuselage aerodynamic force coefficients with incidence angles
Due to very small influence the roll action can be neglected.

2-6 Empennage

The empennage of a helicopter is composed by the horizontal tailplane and the vertical fin. Even if moments produced by the empennage itself are small and negligible, they are at long moment arms from the center of gravity and this produce couples that help a lot to stabilize the vehicle driving. The main action of the horizontal stabilizer is in steady forward flight to generate a trim load that reduces the main rotor fore "aft flapping. The main action of the vertical stabilizer is instead to generate a sideforce and yawing moment serving to reduce the tail rotor thrust requirement. As with the fuselage, the force and moments can be expressed in terms of coefficients that are functions of incidence and sideslip angles. Again a more accurate description can be done with the wind tunnel data but use equations based on these data means have a not general model. For helicopters like the UH-60 for which an adequate literature exists, find wind tunnel approximated functions is not really difficult (see [7]). However for helicopters less "famous" (or in general for civil helicopters) this detailed literature doesn’t exist and hence accurate approximations have to be made. For this reason also in this model wind tunnel approximations are not used and a more general description is chosen. Figure 2-9 shows the arms of the two components of the empennage. \( \alpha_{tp0} \) and \( \beta_{fn0} \) are the zero lift incidence angle on tailplane and zero lift sideslip angle on fin respectively. These two parameters can be fixed or changing with pilot input or automatically in function of the wind but fix them on a value is not an error because for many maneuvers is not necessary to change it. For simplicity in this work both values are set to zero. In the real helicopter \( \beta_{fn0} \) is generally zero while the tail plane can change between 0 and 12 degrees.

Horizontal and vertical stabilizer can be model similarly. Assuming constant the incidence at the tail plane across its span can be written as

\[
\alpha_{tp} = \alpha_{tp0} + \tan^{-1}\left(\frac{w + q_{tp} - k_{\lambda_p} \Omega \lambda_0}{u}\right), \quad u \geq 0
\]

\[
(\alpha_{tp})_{\text{reverse}} = (\alpha_{tp})_{\text{forward}} + \pi, \quad u < 0
\]

The local flow velocity at the tail can be written in the form

\[
\mu_{tp}^2 = \frac{u^2 + (w + q_{tp} - k_{\lambda_p} \Omega \lambda_0)^2}{(\Omega R)^2}
\]
As in the fuselage there is a factor that defines the amplification of the main rotor wake at the tail. Again this value is chosen trying several values. However, due to the smaller dimensions of the tail plane respect to the fuselage, the main rotor wake impinges on the horizontal tail surface only when the wake angle falls between $\chi_1$ and $\chi_2$, given by

$$
\chi_1 = \tan^{-1} \left( \frac{l_{tp} - R}{h_r - h_{tp}} \right), \quad \chi_2 = \tan^{-1} \left( \frac{l_{tp}}{h_r - h_{tp}} \right)
$$

and shown in Figure 2-10

otherwise, $k_{\chi_{tp}}$ can be set to zero.

Taken wind tunnel data of different helicopters, Loftin ([13]) gives a good general approximation of the force coefficient. This approximation is a saturated sin function so that

$$
C_{ztp}(\alpha_{tp}) = a_{0tp} \sin \alpha_{tp}
$$

with saturation in

$$
C_{ztp}(\alpha_{tp}) = \pm C_{ztpl} \sin \alpha_{tp}
$$
Two new values appear in these equations: $\alpha_{tp}$ is the slope of the tail plane lift coefficient curve for small angles of incidence and is typically between 3.5 and 4.5 while the constant limit value $C_{ztpl}$ is approximately 2 for the NACA 0012 aerofoil and seems to be good for this model. For this work precise values are taken again from [7] while for not famous helicopters a tuning must be done.

The local angle of sideslip and velocity (in $x-y$ plane) at the vertical fin may be written in the form

$$\beta_{fn} = \beta_{fn0} + \sin^{-1}\left(\frac{v - r_{fn}l + h_{fn}p}{(\Omega R)\mu_{fn}}\right)$$  \hspace{1cm} (2-91)$$

with

$$\mu_{fn}^2 = \left[\frac{(v - r_{fn}l)^2 + u^2}{(\Omega R)^2}\right]$$  \hspace{1cm} (2-92)$$

where

$$\mu_{fn} = \frac{V_{fn}}{\Omega R}$$  \hspace{1cm} (2-93)$$

A similar approach of that of the tail plane can be used that is a simple analytic function described by a sinusoidal function.

### 2-6-1 Summary of the empennage equations

The equations of fin and tail plane are

$$Y_{fin} = \frac{1}{2} \rho S_{fn} V_{fn}^2 C_{y_{fn}}(\beta_{fn})$$  \hspace{1cm} (2-94)$$

$$Z_{tp} = \frac{1}{2} \rho S_{tp} V_{tp}^2 C_{z_{tp}}(\alpha_{tp})$$  \hspace{1cm} (2-95)$$
\[ L_{fin} = -h_{fn}Y_{fin} \quad (2-96) \]

\[ M_{tp} = -l_{tp}Z_{tp} \quad (2-97) \]

\[ N_{tp} = l_{fn}Y_{fin} \quad (2-98) \]

2-7 Flight control system

As seen in previous sections the pilot drives the helicopter through four commands that are following summarize again

- \( \theta_0 \) = Collective pitch angle given by the collective lever
- \( \theta_{ls}, \theta_{lc} \) = Longitudinal and lateral pitch given by the cyclic
- \( \theta_{0r} \) = Tail rotor collective pitch angle given by the pedals

so what is discussed in this section is how to model the flight control system including the pilot’s controls, the mechanical linkages and the so called control augmentation system (SCAS) that helps the pilot to drive. In particular this last component can be activated or not and for the test that will be shown in chapter 4 it will be off. The scheme of the control system implemented is Figure 2-11

Before describe every component in details some important considerations can be done simply looking the picture

- Commands are coupling by mechanical interlink. In detail the collective lever is interconnected with the longitudinal cyclic and with the pedals and this means that the action of the collective brings the pilot to compensate with pedal and cyclic on the other axes.
- The pitch and the roll actions are coupling and a cyclic mixing is implemented to split best as possible the two actions
- In Figure 2-11 is easy to see the implementation of the SCAS that goes directly to change the input given by the pilot to help the guide.

The general structure of the inputs is the same so that a similar transfer function is used defined as

\[ \theta_i = \frac{\theta_{ip} + \theta_{is}}{1 + \tau s} \quad (2-99) \]

where \( \tau \) is a time constant to describe the lag time of the actuators and chosen of 25 ms and the numerator differentiate the inputs and the subscripts p and a mean \textit{pilot} and \textit{SCAS}
Due to the interaction between its actions the cyclic input is the most complex to describe. The coupling of the pitch and the roll action makes difficult to have pure pitch or pure roll especially when forward speed increases. A single mixing is implemented to try to decouple these actions in the different flight conditions

\[
\begin{bmatrix}
\theta_{1s} \\
\theta_{1c}
\end{bmatrix} =
\begin{bmatrix}
\cos \Psi_f & \sin \Psi_f \\
-\sin \Psi_f & \cos \Psi_f
\end{bmatrix}
\begin{bmatrix}
\theta_{1s}' \\
\theta_{1c}'
\end{bmatrix} \tag{2-100}
\]

where \( \theta_{1s}' \) and \( \theta_{1c}' \) are the real inputs of the pilot written as in (2.95) and the mixing angle \( \Psi_f \) is 0.175 rad for this helicopter. The dynamic of pitch and roll can be written as

\[
\theta_{1s_p} = g_{1s_0} + g_{1s_1} \eta_{1s} + (g_{s_{c_0}} + g_{s_{c_1}} \eta_{1s}) \eta_c \tag{2-101}
\]

\[
\theta_{1c_p} = g_{1c_0} + g_{1c_1} \eta_{1c} + (g_{c_{c_0}} + g_{c_{c_1}} \eta_{1c}) \eta_c \tag{2-102}
\]
where $\eta_{1s}$ and $\eta_{1c}$ are the pilot’s cyclic stick inputs, $\eta_0$ is the pilot’s collective lever input and the various $g$ coefficients as gains and offsets. The $g_1$ coefficients can be expressed as

- $\theta_{1s0}$ - the pitch at zero cyclic stick and zero collective lever
- $\theta_{1s1}$ - the pitch at maximum cyclic stick and zero collective lever
- $\theta_{1s2}$ - the pitch at zero cyclic stick and maximum collective lever
- $\theta_{1s3}$ - the pitch at maximum cyclic stick and maximum collective lever

and

$$
\begin{align*}
g_{1s0} &= \theta_{1s0} \\
g_{1s1} &= \theta_{1s1} - \theta_{1s0} \\
g_{1c0} &= \theta_{1s2} - \theta_{1s2} \\
g_{1c1} &= (\theta_{1s3} - \theta_{1s2}) - (\theta_{1s1} - \theta_{1s0})
\end{align*}
$$

After a conversation with the author of the book [2] on which this work is largely based, was understood that because of there is no mechanical interlink elapses between collective and cyclic, parameters shown above can be set to zero. This important aspect means that for the UH-60 equations (2.97), (2.98) are simply

$$
\begin{align*}
\theta_{1sp} &= (g_{s,c0} + g_{s,c1}\eta_{1s})\eta_c \\
\theta_{1cp} &= (g_{c,c0} + g_{c,c1}\eta_{1c})\eta_c
\end{align*}
$$

Even if is not used in tests done for this work an augmented system is implemented as a proportional stabilizer

$$
\begin{align*}
\theta_{1s} &= k_\theta \theta + k_q q \\
\theta_{1c} &= k_\phi \phi + k_p p
\end{align*}
$$

Main rotor collective and tail rotor collective are implemented in a similar way but in these channels there is no autostabilizer and this characteristic is common for many helicopters and not only for the UH-60. Hence equations are

$$
\begin{align*}
\theta_0T &= g_{T0} + g_{T1}\eta_c T \\
\theta_0 &= g_{c0} + g_{c1}\eta_c
\end{align*}
$$
2-8 Scheme of the model

In the preceding sections individual subsystems have been studied one at a time finding the closed form equation to describe their actions. Figure 2-12 shows a general scheme of the model described.

\[ \dot{x} = F(x, u, t) \]  

where the state vector contains the subsystems shown in Figure 2-12.

\[ x = \{x_f, x_r, x_c\} \]  

\[ x_f = \{u, v, w, p, q, r, \phi, \theta, \psi\} \]
\[ \mathbf{x}_r = \{ \beta_0, \beta_{ac}, \beta_{1s}, \lambda_0, \lambda_{1c}, \lambda_{1s} \} \] (2-113)

\[ \mathbf{x}_c = \{ \theta_0, \theta_{1s}, \theta_{1c}, \theta_0 T \} \] (2-114)

and the input vector is composed of the controls that is

\[ \mathbf{u} = \{ \eta_0, \eta_{1s}, \eta_{1s}, \eta_0 T \} \] (2-115)
Chapter 3

First validation of the dynamic behavior

3-1 Introduction

Chapter 2 has shown how much complex is the helicopter dynamics. Many parameters are present in the equations and some of these need to be tuned. Verify the correctness of the system is not simple due to the difficult to use the model: how closer is to a real helicopter how much difficult is to drive if the driver is not a pilot! Hence, PID controllers are implemented to allow the use of one input at time. A basic environment is developed in Unity to watch the helicopter movements and every subsystem of the model is tested. These steps are fundamental to arrive to the real validation that will be described in chapter 4.

3-2 PID controllers

Because of the model described in chapter 2 is strongly nonlinear without any control action it becomes unstable in few seconds. In a real helicopter the controller is the pilot besides the SAS that however can be switch off or in some model is not present (is not the case of the helicopter of this thesis). Equations written in chapter 2 have shown that is possible to split the helicopter model in four different subsystems but that these subsystems are coupled one each other. In few words the system is MIMO with strongly interconnections between the subsystems. This interconnection makes difficult to drive a helicopter because one input generates more actions (e.g a pitch command produces also roll). A professional pilot knows that the action of the collective needs to be compensated with pedals, that the pitch due to the cyclic is coupled with the roll as the roll with the pitch and so on. However to understand if the dynamic is rightly implemented and all the subsystems give a proper contribution the responses in time of the system have to be seen. But this process is long and is not possible to have a pilot as tester for more than two hours. Moreover many steps and test make a model drivable and so a pilot can really help only when the model is close enough to a real
one. In order to stabilize the helicopter to see the responses in time without the help of a pilot, controllers need to be implemented.

Class of controllers that could be applied to a nonlinear system is very large. Most used in aerospace systems as in many other industrial applications are the Proportional Integrative Derivative (PID) controllers. Figure 3-1 shows the closed loop system in which the PID controller is inserted in series with the system to control.

![Figure 3-1: Closed loop system](image)

The signals shown in figure are

- $r(t)$ is the reference signal
- $y(t)$ is the output signal of the system in retroaction
- $e(t)$ error due to the difference between $r(t)$ and $y(t)$
- $u(t)$ control input

The name PID indicates the three different actions that the controller can do to stabilize the system. Figure 3-2 shows these actions that can be used together or in various combinations (excepted the derivative action alone that intends a predictive action).

![Figure 3-2: PID actions](image)
In Figure 3-2 $K_p$, $K_i$ and $K_d$ are the proportional, integral and derivative gain respectively. The meaning of the different actions is:

- **P** = The proportional term produces an output value that is proportional to the current error value. The tune of the proportional gain is fundamental: if the proportional gain is too high, the system can become unstable, if is too low results in a small output response to a large input error, and a less responsive or less sensitive controller.

- **I** = The integral term accelerates the movement of the process towards setpoint and eliminates the residual steady-state error that occurs with a pure proportional controller. However, since the integral term responds to accumulated errors from the past, it can cause the present value to overshoot the setpoint value.

- **D** = Derivative action predicts system behavior and thus improves settling time and stability of the system. Because of its variable impact on system stability in real-world applications is not often used.

Controllers implemented in this work are *velocity controllers* that means the reference signal is the velocity. Following the different controllers are described in details.

### 3-2-1 Attitude control

The aircraft attitude control involves the roll and the pitch angles $\phi$ and $\theta$. The controllers are implemented to regular the angular velocities $p$ and $q$ that are related with the angles by the noted relations $p = \dot{\phi}$ and $q = \dot{\theta}$. However to obtain a better dynamic response $p$ and $q$ are expressed in function of linear speed $u$ and $v$. Choosing linear speed as reference signals a better response is obtained because small errors in $p$ and $q$ as in $\phi$ and $\theta$ can produce not negligible errors in position. Two specular structures are used for the two controllers:

Roll control:

- PD controller on $v$
- PI controller on $p$

Pitch control:

- PD controller on $u$
- PI controller on $q$

With this configurations the internal model principle is respected so that the error at regime is canceled. The integrator is only in the internal loop and not in the external and this guarantee the stability of the closed loop system. Figure 3-3 and Figure 3-4 show the controllers just described.
3-2-2 Rate-gyro

The rate-gyro control has the task to counter the rotational speed around the yaw axes but without cause less agility to the helicopter. A PID controller is applied as shown in Figure 3-5.

3-2-3 Altitude control

Figure 3-6 shows the PID controller, specular to the one used for the yaw, implemented to control the vertical velocity $w$.

3-2-4 Hover control

The hover is by definition the condition in which all the velocity are equal to zero. This condition is one of the most difficult to get and the difficult increases when the aircraft is big like the UH-60. A pilot wont to small aircraft can find difficult to reach the hover with this helicopter. Because of this reason, the hover is the best condition to test the controllers. Moreover this test is useful for another important reason: test with which values of inputs the hover is reached. As will be explained in chapter 4, inputs are regulated with the help...
of a pilot; however values of input taken from flight data are present in [5] and so a first comparison could be done. Hence all the reference signals \((u,v,w,r)\) are set to zero. Parameters of PID are listed in table Table B-5.

<table>
<thead>
<tr>
<th>Control action</th>
<th>P</th>
<th>I</th>
<th>D</th>
</tr>
</thead>
<tbody>
<tr>
<td>(u)</td>
<td>-0.02</td>
<td>X</td>
<td>-0.2</td>
</tr>
<tr>
<td>(v)</td>
<td>0.1</td>
<td>X</td>
<td>0.2</td>
</tr>
<tr>
<td>(w)</td>
<td>-1</td>
<td>-1</td>
<td>-0.0288</td>
</tr>
<tr>
<td>(p)</td>
<td>-0.5</td>
<td>-0.8</td>
<td>X</td>
</tr>
<tr>
<td>(q)</td>
<td>2</td>
<td>1</td>
<td>X</td>
</tr>
<tr>
<td>(r)</td>
<td>-1</td>
<td>-10</td>
<td>-0.0000025</td>
</tr>
</tbody>
</table>
First validation of the dynamic behavior

Figure 3-7, Figure 3-8 and Figure 3-9 show how both rotational and linear velocities go to zero through the action of the PID. Pictures show also the Euler angles that demonstrate the right position assumed by the aircraft to stay in hover as is possible to compare with any video in internet on this helicopter.

![Linear Velocities Controlled by PID](image1)

![Rotational Velocities Controlled by PID](image2)

Outputs of the controllers are the pitch angles of the blades. In hover values obtained are

- $\theta_0 = 8.1^\circ$
- $\theta_{1c} = 2.2^\circ$
- $\theta_{1s} = -3^\circ$
- $\theta_{0T} = 0.4^\circ$

### 3-3 Unity

The difference between a simulator and a video game is very big. A simulator cannot play by everyone like a video game. However, from the point of view of the visualization they are not
different. The environment is an ambient in which the model can move and this environment can be very poor or extremely detailed as in a video game. Hence, to create the environment for this work, the video game program Unity is used. Unity is a cross-platform game engine which code has to be written in C#. In this work two different environment are used with different aim.

1. A very poor environment used to test the reaction of the model
2. A rich environment with an airport and different delimiters to perform tests of the ADS33

In this part of the work only the first environment is used while the second will be discussed in detail in the next chapter. This simple scheme is shown is Figure 3-10

As is possible to see the environment is composed of simply ground and sky with random different scattered geometric figures used as simple references. Two different cameras are implemented, one external that shows the helicopter as in figure Figure 3-10 and one internal that gives the impression to be a pilot. This second is not useful in this part while the external camera will not used in the second environment.

As is possible to see always in Figure 3-10, the helicopter when touch the ground doesn’t fall but lies on the ground. This aspect is not obtained by unity but made by Matlab and is now detailed.

3-3-1 Ground effect in visualization

The name of this subsection explains that here ground effect doesn’t refer to the different behavior of helicopters close or far from the ground that was already described in previous section. In visualization the ground effect is a reaction of the ground that prevents the fall of the helicopter down over the ground. This effect is following schematically in a simple version

In Figure 3-11 $w^I$ and $\dot{w}^I$ are the $z$ position and the velocity but in inertial system and not in body system. To pass form body to inertial system a matrix $H$ is used defined as
First validation of the dynamic behavior

- **Figure 3-10:** A simple environment made with Unity

- **Figure 3-11:** Ground reaction scheme along z

\[
\begin{bmatrix}
\cos(\psi)\cos(\theta) & \sin(\psi)\cos(\theta) & -\sin(\theta) \\
\cos(\psi)\sin(\theta)\sin(\phi) - \sin(\psi)\cos(\phi) & \cos(\phi)\cos(\psi) + \sin(\psi)\sin(\theta)\sin(\phi) & \cos(\theta)\sin(\phi) \\
\sin(\psi)\sin(\phi) + \sin(\theta)\cos(\phi)\cos(\psi) & \sin(\psi)\sin(\theta)\cos(\phi) - \cos(\psi)\sin(\phi) & \cos(\theta)\cos(\phi)
\end{bmatrix}
\]  

(3-1)

- \( \ddot{w}^I \) is instead the acceleration described by (1.3) but considering simply the Newton law (so that mass and force Z) and the gravity acceleration without effects of the other velocities. These three values are multiplied for three constants and sum. Result pass through a switch that active this action only when coordinate z is positive (that means to be underground). If the action is active it is again converted in body system and goes to add to the other contributions of forces. The action along z is not the only one acting on the ground. To obtain the effect of a body that doesn’t move torsional coefficient are adding as damping and stiffness to the rotational velocity. Moreover damping coefficients are also add to the velocities along x and y. The complete simulink scheme is shown in Figure 3-12
3-3-2 Force and moments test

Since the controllers are tested they are used to test the actions of all the subsystems. In fact data tables with exactly the values of forces generated by the single components are not founded in literature and so, even if is possible to check with plots the right direction of acting of every force, is quite impossible understand if the contribution of one action is too strong or too small. In principle this trivial distinction is possible to use to start

- **Main rotor**: has the most contribution in every action but in the yaw
- **Tail rotor**: controls the yaw rate
- **Empennage**: gives stability to the system and helps to drive
- **Fuselage**: smooths the movements

So what is possible to do with the help of controllers without be a pilot is to control in hover every input but not the one to test, and in the channel free give increasing input till the controllers are not more able to maintain the aircraft in hover. This test is performed using before only the main rotor connected to a mass, after adding the fuselage, the tail and the empennage one at time. To see if the action of the single component acts always in the right direction and to see the magnitude of this component arrows are added in unity as shown in Figure 3-13. An example of the test just described is shown in Figure 3-14. In that test the only all the components were active and the arrows represent the action of the fin while every input are controlled by PID but the pedals. So, giving an action on the right, the fin produce a reaction to the left. Because of the action of the fin is more or less only action in the yaw, the magnitude on Y (arrow in the middle) is less than the magnitude on N (arrow in the fin).
First validation of the dynamic behavior

Figure 3-12: Simulink scheme of ground reactions
Figure 3-13: Unity desktop with the arrows added to the helicopter

Figure 3-14: Test with arrows
4-1 Introduction

In the previous chapter forces and moments acting on the model were tested to verify the right action. Now is necessary validify the model comparing its behavior with the one of the real helicopter. The Aeronautical Design Standard-33 (ADS-33) is the standard used with military helicopters like the UH-60. Validation in frequency and time will be done and results will be shown. This is the fundamental step before to test the model with a pilot.

4-2 Aeronautical Design Standard

In 1982 the U.S. Army Aeroflightdynamics Directorate (AFDD) began development of a new handling qualities specification for military rotorcraft. This development effort resulted in the U.S. Army’s initial Aeronautical Design Standard-33, *Handling Qualities Requirements for Military Rotorcraft* published in 1987. From the first version, many modifications were done till arrived to the version published in 1996 (and used in this work) with the name of ADS-33D-PRF that includes utility helicopters and and test for military helicopters with external slung loads.

By definition of Harper and Cooper, the handling qualities are *those qualities or characteristics of an aircraft that govern the ease and precision with which a pilot is able to perform the task required in support of an aircraft role*. The translation of this sentence in actions are the so called MTEs (Mission Task Elements) so that some maneuvers that pilot has to perform. However before arrive to the MTEs is necessary to understand the behavior of the aircraft. Two kind of test must be done to verify the responses of the system, test in tim domain and test in frequency domain. Next two sections will show in details how obtain these responses. In few words what is done is perturb an equilibrium position with doublet inputs or sweeping sinusoids and check the responses in time and in frequency.
4-3 Attitude quickness test

First response analyzed is that in time and called test of quickness. This test can be seen as the rapidity of a rotorcraft to change its attitude. The attitude quickness parameter is stated in the ADS-33D and its definition is

*The required attitude changes shall be made as rapidly as possible from one steady attitude to another without significant reversals in the sign of the cockpit input relative to the trim position.*

The parameter is defined as the ratio of peak attitude rate to change of attitude, which for the roll would be peak roll rate to corresponding change in roll angle. This definition is exactly the same for the pitch axes, the other channel tested in this way. The ability to generate rolling and pitching moments about the aircraft’s centre of gravity serves two purposes. First, to enable the pilot to trim out residual moments from the fuselage, empennage and tail rotor, e.g., repositioning sidestep as longitudinal step or pure hover. Second, so that the rotor thrust vector can be reoriented to manoeuvre in the lateral plane or in the longitudinal plane. Now first of all is necessary to understand how to obtain the ratio of peak. As shown in Figure 4-1, the response to a step input (in figure a lateral input is shown) is an exponential growth to a steady-state rate. To derive a value for attitude quickness is considered the response to a pulse input of duration $t_1$ which leads to a discrete amplitude change $\Delta \phi$.

![Figure 4-1: Response to pulse lateral cyclic input](image)

From these two parameters the ratio of peak is obtained as
4-3 Attitude quickness test

\[
\text{attitude quickness} = \frac{p_{pk}}{\Delta \phi}
\]  

(4-1)

This value is not the only one necessary to obtain the response in time. The value on the abscissa of the plot is the one shown in Figure 4-2 and is the minimum peak with opposite bending after the end of the step input.

\[\Delta \phi\]

\[\Delta \phi_{pk}\]

\[\Delta \phi_{min}\]

\[\text{time}\]

**Figure 4-2:** Definition of the minimum $\Delta \phi$

This parameter makes the test difficult. In fact to see the quickness of the system is not necessary change the attitude and stay after in the new position but could be enough give the input and come back to the original position or close to as shown in Figure 4-3a. However in this case the minimum peak should be always really small and there should be not compliance in the results. For this reason is necessary not return to the initial trim position following the pulse as shown in Figure 4-3b.

Figure 4-3 shows inputs given by professional pilots with the same model of helicopter treated in this work.

Now, replicate this without be a pilot is impossible and require a lot of time of training. At the same time was not possible call a pilot to have an entire day to do all the test necessary. Hence the solution found was to use the controllers explained in the previous chapter to stabilize the helicopter in hover in all the axes but the one to test. Moreover, as done in the test with the real helicopter [14], the control system (SAS) is active all the time. With these two helps is possible to replicate the maneuver that is to give an input of the same amplitude of the real helicopter and maintain the attitude. Figure 4-4a shows again the input given by a real pilot and taken from [14] compared with input given using a joystick in the environment seen in chapter 3 according to what written before (Figure 4-4b).

Master of Science Thesis  
Carlo Andrea Gerboni
As is possible to see in Figure 4-4 giving an input close to the one given using the real helicopter the responses of the system, in terms of rate and angle, are almost the same. The input shown is the one to obtain the left roll response and is the only one for which the real data input was found. The other three input (right roll, pitch up and pitch down) are given with the same variation in percentage. Figure 4-5 and Figure 4-7 show results obtained with the model while Figure 4-6 and Figure 4-8 are results obtained with the real helicopter. Many pilots performed test with the real UH-60 and this is the reason why many results are present in Figure 4-6 and Figure 4-8.

Is possible to see that the results are very good. Not only the model ensues in the level 1 of the ADS-33 but the ratio of peak has the same values of the real test. These results show that the quickness of the aircraft is absolutely comparable with the real UH-60 and this is the first step to speculate that is possible to replicate the ADS-33 MTEs using the model in a flight simulator.

### 4-4 Frequency response

By definition

the frequency response is the quantitative measure of the output spectrum of a system or device in response to a stimulus, and is used to characterise the dynamics of the system. It is
First of all is necessary to understand which quantities are required for the ADS-33 criteria. The two parameters are the bandwidth and the phase delay. The bandwidth parameter is defined in Figure 4-9 as the lesser of two frequencies, the phase-limited or gain-limited bandwidth, derived from the phase and gain of the frequency response of attitude to pilots cyclic control. The phase bandwidth is given by the frequency at which the phase is 135°, i.e., the attitude lags behind the control by 135°.

The gain bandwidth is given by the frequency at which the gain function has increased by 6 dB relative to the gain when the phase is 180°. The 180° phase reference is significant because it represents a potential stability boundary for closedloop tracking control by the pilot. The other parameter used for the ADS-33 is the phase delay. For more general high-order dynamics, the phase delay has to be computed as an independent measure of handling, since configurations with markedly different phase slope can have the same bandwidth. A different phase slope even if the bandwidth is the same or is very similar can rate an helicopter with a level 2 or 3 instead of 1. A mathematical definition of the phase delay is

\[ \tau_p = \frac{\Delta \Phi_{2\omega_{180}}}{57.3 \times 2\omega_{180}} \]  

(4-2)

where \( \Delta \Phi_{2\omega_{180}} \) is the phase change between \( 2\omega_{180} \) and \( \omega_{180} \). The phase delay is therefore related to the slope of the phase between the crossover frequency and \( 2\omega_{180} \).

An example of the ADS-33 boundaries for this test based on the two parameters just defined is shown in Figure 4-10.

The second point now is to understand how to obtain the frequency response of the model that is the Bode diagram. First of all, as for the quickness test all the test are performed considering the control system (SAS) active for safety reason of the pilot. Now having the
nonlinear model the most reasonable method to do the frequency analysis is to linearize the model around the condition considered and from the linearized model look the Bode diagram. The other possibility is to apply the identification method exciting the system with a sweeping sinusoid as input. Due to the fact that the frequency responses of the UH-60 were not found in literature, results of both methods were analyzed in terms of ADS-33 criteria. Hence, from the Bode diagrams the bandwidth and the phase delay were taken and compared with the ones of the real helicopter. Results obtained have shown that to have comparable responses with real data the linearization is not usable. The reason is that the real data results are taken from tests with pilot-in-the-loop while using the linearized model the pilot is not considered. In terms of phase delay in high frequencies this doesn’t cause modification but in terms of bandwidth the pilot-in-the-loop displaces forward the cut-off frequency as in the real data results.  

Before to show the results is necessary explain how to execute the identification. As already said the input is a sweeping sinusoid as the one shown in Figure 4-11 that has a $\omega_{\text{min}} = 0.3 \text{rad/sec}$ as minimum frequency and $\omega_{\text{min}} = 12 \text{rad/sec}$ as maximum.  

This input is given in the axes that we want to test (in this case not only roll and pitch but also the yaw) while the other are manually controlled trying to stay in hover condition for all the duration of the sweep. To excite the system a small amplitude input is adequate and so to control the other inputs is not really difficult. In this way the pilot is in-the-loop as required for this test and the channel of interest is adequately excited.  

The identification process is done using a yet developed method [15] based on Tischler identification method [16]. In this method the contribution of the other inputs are subtracted and the response is valid only when the coherence function is high and the crosscorrelation between the main channel of interest and the others is low.
Figure 4-6: Roll quickness real helicopter response

Figure ??, Figure ?? and Figure ?? show the results obtained. As is possible to see the bandwidth is the same for all the inputs and this is the final (and probably the most important too) validation that the behavior of the model is the one of an UH-60. At the same time the model shows a different delay of the response in the roll and in the pitch already seen with the linearization method. Several reasons could produce this different delay e.g., some wrong values allowed to the unknown parameters, the absence of the rotor powerplant or the use of the actuation theory instead of the blade theory.
Validation with the ADS-33

Figure 4-7: Pitch quickness model response

Figure 4-8: Pitch quickness real helicopter response
Figure 4-9: Definition of the parameters obtained from the frequency response

Figure 4-10: ADS-33 requirements for the frequency responses
Figure 4-11: Sweeping sinusoid used for the identification

ADS33 PITCH RESPONSE IN HOVER

ADS33 ROLL RESPONSE IN HOVER
4.4 Frequency response

ADS33 YAW RESPONSE IN HOVER

Level 3
Level 2
Level 1

model
real data

ω_{\text{BW}} \psi (\text{rad/s})
τ \psi (s)

ω_{\text{IMF}} (\text{rad/s})

0 1 2 3 4 5
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1

Master of Science Thesis
Carlo Andrea Gerboni
Chapter 5

Test with a pilot in a simulator without motion

5-1 Introduction

All test done in chapter 3 and 4 have shown that the model reproduces the behavior of the UH-60. However the comparison with the ADS-33 were done only in hover condition but not in other conditions. This is an optimum starting point being the hover condition the most difficult condition to reach for a pilot but is not enough to say that the pilot fells the model as an helicopter. Moreover the doubt parameters influnce many flight conditions and only a person with experience of drive can understand if something is going wrong. In this chapter a pilot rating scale is used to rate maneouvers performed by a pilot in a simulator without motion.

5-2 Pilot rating scale

Despite this project born to make a model that can be used in simulator with motion the first step is to test it in a simulator without motion. There are several reasons to do this previous step:

• is possible to reply all the different flight conditions without limitations caused by the workload of the simulator
• is really possible to test the model excluding possibly errors due to the communication between the simulator and the simulink model
• is fast change some parameters that are in doubt and test again with the value changed
• the pilot can stop and go the test everytime he wants simply pressing a button
The simulator and the environment will be described in the next section. In this want is described is the pilot rating scale used. For simulator with motion the Pilot rating scale used is the famous Harper-Cooper that born to be used with the ADS-33 MTEs. However this rating scale cannot be used for simulator without motion because the motion fills one part of the test itself. So another test needs to be used and the one chosen is the Pilot Rating Scale suggested by the IAI (Israel Aircraft Industries). This scale is used by Tischler and others to test their simulators [17]. The entire set of questions are shown in Figure 5-1

As is possible to see the questions are divided in three groups. The first group is the primary helicopter response that rates the action of the main input used to performed the maneuver chosen. For example, if the maneuver that the pilot has to perform is the take-off the primary response is the action of the collective related with movement seen along the vertical axes. The secondary helicopter response is consequently the evaluation of the other inputs that is the compensation that the pilot has to do with the other inputs. Using the same example of the take-off, the secondary response is the compensation that the pilot does with the pedals due to the turning produced by the collective. In both this two groups, the questions evaluate what the pilot sees as the response of the controls. Due to the lack of motion the environment can influence a lot the response of the pilot and some maneuvers could be very difficult to perform. So is necessary distinguish if a low rate is caused by dissimilarities between the
model and the helicopter or if the problem is the visualization. For this reason the third group is inserted and comments to the pilot are asked at the end of the maneuver. The pilot rating scale for the helicopter responses goes from 0 to 6, where 6 represents the highest similarity. The difficulty of execution scale goes from 0 to 5, where 5 represents the highest similarity in the difficulty of execution.

5-3 Fixed base simulator and the environment

The fixed-base simulator is the Panolab shown in Figure 5-2

![Panolab room](image)

This laboratory is equipped with seven projectors that show in a large screen the environment made in Unity. The simulink scheme runs in real time using a xpc.

The environment is an airport surrounded by mountains. In the airport there are different markers (bars, sphere, square). These elements are positioned according to some maneuvers of the ADS-33. In details the two MTEs defined by the elements in the scheme are:

- Vertical remask (Figure 5-3)
- Acceleration-deceleration maneuver (Figure 5-4)

Figure 5-5 shows the environment with the markers. A description of them is added. In subsequent section the ADS-33 maneuvers performed will be described.

1. The starting point is a square. The helicopter starts in the middle of the square.
2. In front of the square there is a bar long 22.8 m. At the end there is a sphere attached to the bar. Another sphere is situated at 7.6 m. These two spheres are reference point for hover and vertical maneuvers.
3. Another square with the same dimension is located parallel at 91 m on the right. Along all the path several bars are located to help the pilot to maintain the altitude. On the ground coloured small squares are positioned to define the limits of the MTEs. This path is used for lateral maneuvers.

4. In front of the square another square is located 91 m forward. Again lateral bars are located along all the path to help the pilot with the altitude. On the ground coloured small squares are positioned to define the limits of the MTEs. This path is used for longitudinal displacement.

The pilot drives the helicopter in this environment using exactly the same commands of the aircraft. Collective, cyclic and pedals are provided by Wittenstein shown in Figure 5-6.

Before start with the maneuver the ranges of the inputs are checked by the pilot. The session with the pilot are divided in two parts: in the first four basic maneuvers are performed in such
5-4 Maneuvers evaluated with the PRS IAI

Using the pilot rating scale described in section (4-2) four maneuvers are asked to perform to the pilot. These maneuvers cover all the six degrees of freedom of the aircraft. To be performed the pilot has to use all the commands paying particular attention on one of them per maneuver. The maneuvers chosen allow to verify also the coupling between the inputs to rate it in the secondary response section of the test. For this test there are no limitation in time: the pilot has to drive normally and time is not one of the parameter to test. After each maneuver the entire set of questions is asked to the pilot. Now for every maneuver will be described the maneuver itself, the rating given by the pilot and his comments. The pilot has more than 110 hours of flight with a real helicopter with more than 700 takeoff and landings. He has also experiences with simulators of flight. 

Note: the pedals had a mechanical problems that makes them too sensible. Use them was difficult for the pilot.

1. TAKEOFF AND HOVER

- **Description of the maneuver**
  The maneuver starts with the helicopter in the middle of the first square. The pilot has to takeoff and move up till the end of the bar staying in hover at 22.8 m (position marked with a sphere as explained before). When the hover condition is reached, starts a vertical descend till the first sphere located at 7.6 m and stay in a new hover.

- **Rating (expressed as average)**
58 Test with a pilot in a simulator without motion

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2. LATERAL DISPLACEMENT

- **Description of the maneuver**
  First of all takeoff and hover at 7.6 m. The maneuver starts in this point. The pilot has to perform a lateral displacement of 91 m and stay in hover upon the square.

- **Rating (expressed as average)**
  - Primary response = 3
  - Secondary response = 3.5
  - Difficulty of execution = 2.5

- **Comments**
  The secondary response is difficult to compensate due to the sensitivity of the pedals. In this maneuver is very difficult to perceive the secondary response due to the lack of motion. The action of the lateral cyclic is very good but the pedal penalize the evaluation.

3. FORWARD AND BACKWARD MOVEMENTS

- **Description of the maneuver**
This maneuver starts where the previous ends. From the second square at the height of 7.6 m move forward for 91 m till the other square. There hover trying to stay at the same altitude of the start during all the maneuver. Reached the hover move backward till the first square.

- **Rating (expressed as average)**
  - Primary response = 3.8
  - Secondary response = 3.8
  - Difficulty of execution = 2.5

- **Comments**
  *The overall evaluation is very good. Again the lack of motion makes difficult to maintain the altitude during the backward displacement.*

4. **360° TURNING**

- **Description of the maneuver**
  In the airport there is a tower. This maneuver consists of a 360° turning around this tower. The pedals are the first control, the lateral cyclic the second.

- **Rating (expressed as average)**
  - Primary response = 4
  - Secondary response = 4
  - Difficulty of execution = 3

- **Comments**
  *This maneuver certifies that the problem of the pedals is mechanical and not of the model. Is possible to turn using only the pedals and the coupling is very good.*

5. **OVERALL EVALUATION**

The overall evaluation of the model is very good and considered "similar" to the real helicopter. The lack of motion is a too important factor for the pilot but his final comment gratifies the work:

*Better than the other simulators I’ve tested in the past. It seems to drive a helicopter, the experience of flight is very close to the real one.*

5-5 **ADS-33 vertical remask**

Test shown above prove the ability of the model to be used in a simulator without motion. The cybermotion simulator in which this model will be tested was busy for all the time of the work done for this thesis and it will be tested after the publication of this work. However is interesting understand if the pilot is able to perform difficult task with this model yet without motion. If the answer is yes the adding of the motion will help the execution of the ADS-33 test that will be rated by the Harper-Cooper scale.

As said in section (4-2) the markers in the environment are displaced according to two MTEs. There are two reasons why these two maneuvers are chosen:
• All the basic maneuvers can be performed with this environment as seen before

• The MPI Cybermotion Simulator has a lateral rail and so the workspace for the lateral movement is bigger than the longitudinal.

The description of the maneuver and the performance desired are shown in Figure 5-7 that is the official page of the ADS-33 document.

3.11.10 Vertical Remask

a. Objectives.
   • Check ability to accomplish an aggressive vertical descent close to the ground.
   • Check ability to combine vertical and lateral aggressive maneuvering as required to evade enemy fire if observed during a bob-up.

b. Description of maneuver. From a stabilized hover at 75 ft, remask vertically to an altitude below 25 ft. Then rapidly displace the rotorcraft laterally 300 ft and stabilize at a new hover position. During the vertical remask simulate deploying rotorcraft survivability equipment as appropriate. Accomplish the maneuver to the left and to the right.

c. Description of test course. The test course should include markers to denote the desired and adequate performance related to position during the vertical descent and final stabilized hover. It may also be desirable to include a vertical reference to provide cues related to the 25 ft altitude reference. This maneuver assumes that the pilot is remasking behind some object, and such an altitude reference should therefore be available. A suggested test course for this maneuver is shown in Figure 26.

d. Performance standards.

<table>
<thead>
<tr>
<th>Performance – Vertical Remask</th>
<th>Desired</th>
<th>Adequate</th>
</tr>
</thead>
<tbody>
<tr>
<td>• Achieve an altitude of X or less within 6 seconds of initiating the maneuver.</td>
<td>25 ft</td>
<td>NA</td>
</tr>
<tr>
<td>• During initial stabilized hover, vertical descent, and final stabilized hover, maintain longitudinal and lateral position within ±X ft of a reference point on the ground.</td>
<td>8 ft</td>
<td>12 ft</td>
</tr>
<tr>
<td>• Maintain altitude after remask and during displacement within X ft:</td>
<td>±10 ft</td>
<td>+10 and -15 ft</td>
</tr>
<tr>
<td>• Maintain lateral ground track within ±X ft:</td>
<td>10 ft</td>
<td>±15 ft</td>
</tr>
<tr>
<td>• Maintain heading within ±X deg.</td>
<td>10 deg</td>
<td>±15 deg</td>
</tr>
<tr>
<td>• Achieve a stabilized hover within X sec after reaching the final hover position.</td>
<td>5 sec</td>
<td>10 sec</td>
</tr>
<tr>
<td>• Achieve the final stabilized hover within X sec of initiating the maneuver.</td>
<td>15 sec</td>
<td>25 sec</td>
</tr>
</tbody>
</table>

Figure 5-7: ADS-33 vertical remsk maneuver description

As is possible to note in Figure 5-7 the performance could be adequate or desired. Due to the difficulties perceived by the pilot because of the lack of motion, what results aspected were at least adequate or close by. Instead results are excellent for almost all the performances:

• **Maintain longitudinal position within 3.7 m (2.4 m)** Desired performance in the initial hover and in the vertical descend while only in the last hover was close by to be adequate

• **Maintain lateral position within 3.7 m (2.4 m)** Close to zero all the time, the best performance of all.

• **Maintain altitude after remask within 4.6 m (4.6 m)** This performance is not adequate but only for a meter. This is probably the most difficult performance to respect due to the lack of motion and the sensitivity of the pedal and in fact was the cause of a less evaluation during the vertical displacement. In this contest this result is very good and suggests that with the motion the pilot can reach high performances.
• **Maintain lateral ground track within 4.6 m (3 m)** Other performance largely achieved. The response is desired all the time except for the beginning when is adequate.

• **Maintain heading within 15°** This performance is desired in the vertical remask, adequate during the lateral displacement and not good only in the final hover but the sensitivity of the pedals doesn’t help.

• **Achieve a stabilized hover after 10 sec (5 sec)** Close to be adequate (13 sec)

• **Accomplish the entire maneuver within 25 sec (15 sec)** In this case too the performance is close to be adequate (31 sec)
Test with a pilot in a simulator without motion
Future steps

6-1 MPI Cybermotion Simulator

As said in the introduction, few works are developed considering all the components of a real helicopter. The majority of the work are used only in simulation and don’t need a high level of precision. However models used in simulator without motion are few. The only work in which a model is tested with a proper pilot rating scale as in this work is the already cited [17]. If the literature of model without motion is little, literature of model tested in simulator with motion is almost absent. Simulators with motion are few in Europe. The most famous are the SIMONA simulator in Delft Figure 6-1 and the HELIFLIGHT in Liverpool Figure 6-2.

Papers based on these simulators are available but they describe only how these simulators work or which kind of test are done but no information about models used are available. About the simulator in Delft some papers are found but model used have really simplified dynamics, often simply a double integrator dynamics. About the simulator in Liverpool is easy to speculate that the model used is similar to the one developed in this work and based on the theory of Professor Padfield but works are not readily available.

The next step of this work will be use the model in the MPI Cybermotion simulator Figure 6-3 Actually the model is already tested in a simulator of the robot and result is good for lateral maneuver. In fact using the lateral rail for lateral displacement the movement can be reproduced without washout filters. However for longitudinal movements as for large lateral displacement an accurate work on the washout filters will be required.

The first test will be reproduce the same maneuvers performed in the simulation without motion and compared the results with the ones that will be obtained with the Harper-Cooper rating scale. To use the Harper-Cooper rating scale and obtain consistent data many pilots must test the model. The second test could be choose other MTEs over the vertical remask and this involves the making of different environments.
Figure 6-1: SIMONA simulator in Delft

Figure 6-2: HELIFLIGHT simulator in Liverpool
Figure 6-3: MPI Cybermotion Simulator
Appendix A

A-1 Reference systems

In aerospace systems different reference frames are used and sometimes some misunderstanding could append. In particular is necessary to define the reference axes frame with which all measurements are given and the body axes frame in which act the equations derived in chapter 2. As is possible to see in Figure A-1 and Figure A-2 in the two different frames have an opposite direction of $x$ and $z$ axes. Now, following the approach used by [12] from which all the data are taken, measurement are given in the reference axes frames but to be used in force and moment equations must be changed in sign.

![Reference axes frame](image)

**Figure A-1:** Reference axes frame
Figure A-2: Body axes frame

To not make confusion of sign all the distances that are present in equations are count following and are expressed in body reference frame

- **Main Rotor**

  \[ x_h = -(x_{h\text{sta}} - c_{g\text{sta}}) \]

  \[ h_R = -(z_{h\text{wat}} - c_{g\text{wat}}) \]

- **Tail rotor**

  \[ l_T = -(l_{T\text{sta}} - c_{g\text{sta}}) \]

  \[ h_T = -(h_{T\text{wat}} - c_{g\text{wat}}) \]

- **Empennage**

  \[ l_{fn} = -(l_{f\text{nsta}} - c_{g\text{sta}}) \]

  \[ h_{fn} = -(h_{f\text{nwat}} - c_{g\text{wat}}) \]

  \[ l_{tp} = -(l_{p\text{sta}} - c_{g\text{sta}}) \]
\[ h_{tp} = -(h_{tp_{wat}} - cg_{wat}) \]

The sign in the equations of moments are found with the mechanical equation

\[ \vec{M} = \vec{p} \times \vec{F} \quad (A-1) \]

where \( \vec{p} \) is the arm and \( \vec{F} \) is the vector of the forces. The development of this equation is

\[
\begin{bmatrix}
M_x \\
M_y \\
M_z
\end{bmatrix} =
\begin{bmatrix}
p_x \\
p_y \\
p_z
\end{bmatrix} \times
\begin{bmatrix}
F_x \\
F_y \\
F_z
\end{bmatrix} =
\begin{bmatrix}
p_2F_3 - p_3F_2 \\
p_3F_1 - p_1F_3 \\
p_1F_2 - p_2F_1
\end{bmatrix} \quad (A-2)
\]

### A-2 Angular coordinates of the aircraft

This section and the next one are entirely taken by [2].

The helicopter fuselage can take up a new position by rotations about three independent directions. The new position is not unique, since the finite orientations are not vector quantities, and the rotation sequence is not permutable. The standard sequence used in flight dynamics is yaw, \( \psi \), pitch, \( \theta \), and roll, \( \phi \), as illustrated in Figure A-3. We can consider the initial position as a quite general one and the fuselage is first rotated about the z-axis (unit vector \( k_0 \)) through the angle \( \psi \), (yaw). The unit vectors in the rotated frame can be related to those in the original frame by the transformation \( \Psi \), i.e.,

\[
\begin{bmatrix}
i_1 \\
j_1 \\
k_0
\end{bmatrix} =
\begin{bmatrix}
\cos \psi & \sin \psi & 0 \\
-\sin \psi & \cos \psi & 0 \\
0 & 0 & 1
\end{bmatrix}
\begin{bmatrix}
i_0 \\
j_0 \\
k_0
\end{bmatrix} \quad \text{or} \quad \{b\} = \Psi \{a\} \quad (A-3)
\]

Next, the fuselage is rotated about the new y-axis (unit vector \( j_1 \)) through the (pitch) angle \( \theta \), i.e.,

\[
\begin{bmatrix}
i_2 \\
j_2 \\
k_1
\end{bmatrix} =
\begin{bmatrix}
\cos \theta & 0 & -\sin \theta \\
0 & 1 & 0 \\
\sin \theta & 0 & \cos \theta
\end{bmatrix}
\begin{bmatrix}
i_1 \\
j_1 \\
k_0
\end{bmatrix} \quad \text{or} \quad \{c\} = \Theta \{b\} \quad (A-4)
\]

Finally, the rotation is about the x-axis (roll), through the angle \( \phi \), i.e.,

\[
\begin{bmatrix}
i_2 \\
j_2 \\
k_2
\end{bmatrix} =
\begin{bmatrix}
1 & 0 & 0 \\
0 & \cos \phi & \sin \phi \\
0 & -\sin \phi & \cos \phi
\end{bmatrix}
\begin{bmatrix}
i_2 \\
j_1 \\
k_1
\end{bmatrix} \quad \text{or} \quad \{d\} = \Phi \{c\} \quad (A-5)
\]

Any vector, \( \mathbf{d} \), in the new axes system can therefore be related to the components in the original system by the relationship

\[ \{d\} = \Phi \Theta \Psi \{a\} = \Gamma \{a\} \quad (A-6) \]
Appendix A

Since all the transformation matrices are themselves orthogonal, i.e.,

\[ \Phi^T = \Phi^{-1}, \quad \Theta^T = \Theta^{-1}, \quad \Psi^T = \Psi^{-1} \]  \hspace{1cm} (A-7)

the product is also orthogonal, hence

\[ \Gamma^T = \Gamma^{-1} \]  \hspace{1cm} (A-8)

\[ \begin{align*}
    a_{xg} &= -g \sin \theta \\
    a_{yg} &= g \cos \theta \sin \phi \\
    a_{zg} &= g \cos \theta \cos \phi
\end{align*} \]  \hspace{1cm} (A-9)

Figure A-3: The fuselage Euler angles: (a) yaw; (b) pitch; (c) roll

A-3 Components of gravitational acceleration along the aircraft axes

The relationships derived above are particularly important in flight dynamics as the gravitational components appear in the equations of motion in terms of the Euler angles while the aerodynamic forces are referenced directly to the fuselage angular motion. We assume for helicopter flight dynamics that the gravitational force always acts in the vertical sense and the components in the fuselage-fixed axes are therefore easily obtained with reference to the transformation matrix given by \( \Gamma \). The gravitational acceleration components along the fuselage x, y and z axes can therefore be written in terms of the Euler roll and pitch angles as

\[ \begin{align*}
    a_{xg} &= -g \sin \theta \\
    a_{yg} &= g \cos \theta \sin \phi \\
    a_{zg} &= g \cos \theta \cos \phi
\end{align*} \]  \hspace{1cm} (A-9)

A-4 The rotor system

In chapter 2, section 1-3 a mathematical model of the main rotor is developed and three different reference systems appear during the discussion: the hub system, the hub-wind system
and the body system. The reason why this is done is now explained.

Figure A-4 illustrates the hub reference axes, with the $x$ and $y$ directions oriented parallel to the fuselage axes centred at the centre of mass. The $z$ direction is directed downwards along the rotor shaft, which, in turn, is tilted forward relative to the fuselage $z$-axis by an angle $\gamma_s$. The zero azimuth position is conventionally at the rear of the disc as shown in the figure, with the positive rotation anticlockwise when viewed from above, i.e., in the negative sense about the $z$-axis. Positive flapping is upwards. The positive $y$ and $z$ directions are such that the blade and hub systems align when the flapping is zero and the azimuth angle is $180^\circ$.

The hub-wind system is a non rotating hub system which is aligned with the resultant velocity in the plane of the rotor disc and is really convenient to express everything in this system. To the logical chain is

$$ hub \rightarrow hub-\text{wind} \rightarrow body $$

The hub velocity components in the hub reference system are related to the velocities of the centre of mass, $u$, $v$ and $w$ through the transformation

$$
\begin{bmatrix}
u_h \\
v_h \\
w_h
\end{bmatrix} =
\begin{bmatrix}
cos\gamma_s & 0 & sin\gamma_s \\
0 & 1 & 0 \\
-sin\gamma_s & 0 & cos\gamma_s
\end{bmatrix}
\begin{bmatrix}
u - qh_R \\
v + ph_R + rx_h \\
w - qx_h
\end{bmatrix}
$$

(A-10)

while to pass linear and rotational velocities from the hub system to the hub-wind system the transformation are

$$
\begin{align*}
u_{hw} &= (u_h + v_h)^{\frac{1}{2}} \\
v_{hw} &= 0 \\
w_{hw} &= w_h
\end{align*}
$$

(A-11)
\[
\begin{bmatrix}
  p_{hw} \\
  q_{hw}
\end{bmatrix} = \begin{bmatrix}
  \cos \psi & \sin \psi \\
  -\sin \psi & \cos \psi
\end{bmatrix} \begin{bmatrix}
  p \\
  q
\end{bmatrix}
\]

(A-12)

\[r_{hw} = r + \dot{\psi}_w\]

(A-13)

where the rotor sideslip angle \( \psi_w \) is defined by the expressions

\[
\cos \psi_w = \frac{u_h}{\sqrt{u_h^2 + v_h^2}}, \quad \sin \psi_w = \frac{v_h}{\sqrt{u_h^2 + v_h^2}}
\]

(A-14)
Appendix B

Helicopter Parameters

All these data are taken from [12] and [18] and are given in the reference axes.

Table B-1: Aircraft mass and inertia

<table>
<thead>
<tr>
<th>Description</th>
<th>Symbol</th>
<th>UH-60 value</th>
<th>Units</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mass of the helicopter</td>
<td>$M_a$</td>
<td>7438</td>
<td>kg</td>
</tr>
<tr>
<td>Aircraft roll inertia</td>
<td>$I_{xx}$</td>
<td>7632</td>
<td>$kg\ m^2$</td>
</tr>
<tr>
<td>Aircraft pitch inertia</td>
<td>$I_{yy}$</td>
<td>54233</td>
<td>$kg\ m^2$</td>
</tr>
<tr>
<td>Aircraft yaw inertia</td>
<td>$I_{zz}$</td>
<td>50436</td>
<td>$kg\ m^2$</td>
</tr>
<tr>
<td>Aircraft cross product of inertia</td>
<td>$I_{xz}$</td>
<td>2264</td>
<td>$kg\ m^2$</td>
</tr>
</tbody>
</table>

Figure B-1: UH-60 dimensions
### Table B-2: Center of gravity position

<table>
<thead>
<tr>
<th>Description</th>
<th>Symbol</th>
<th>UH-60 value</th>
<th>Units</th>
</tr>
</thead>
<tbody>
<tr>
<td>Center of gravity stationline</td>
<td>$cg_{sta}$</td>
<td>9.2</td>
<td>m</td>
</tr>
<tr>
<td>Center of gravity waterline</td>
<td>$cg_{wat}$</td>
<td>6.3</td>
<td>m</td>
</tr>
<tr>
<td>Center of gravity buttline</td>
<td>$cg_{but}$</td>
<td>0</td>
<td>m</td>
</tr>
</tbody>
</table>

### Table B-3: Main rotor

<table>
<thead>
<tr>
<th>Description</th>
<th>Symbol</th>
<th>UH-60 value</th>
<th>Units</th>
</tr>
</thead>
<tbody>
<tr>
<td>Main rotor speed</td>
<td>$\Omega$</td>
<td>27</td>
<td>rad/sec</td>
</tr>
<tr>
<td>Main rotor blade radius</td>
<td>$R$</td>
<td>8.2</td>
<td>m</td>
</tr>
<tr>
<td>Blade lift curve slope</td>
<td>$a_0$</td>
<td>5.73</td>
<td>rad$^{-1}$</td>
</tr>
<tr>
<td>Main rotor solidity</td>
<td>$s$</td>
<td>0.08210</td>
<td>-</td>
</tr>
<tr>
<td>Rotor shaft forward tilt</td>
<td>$\gamma_s$</td>
<td>0.05236</td>
<td>rad</td>
</tr>
<tr>
<td>Max rotor thrust coefficient</td>
<td>$C_T$</td>
<td>0.1846</td>
<td>-</td>
</tr>
<tr>
<td>Number of blades</td>
<td>$b$</td>
<td>4</td>
<td>-</td>
</tr>
<tr>
<td>Blade Lock number</td>
<td>$\gamma$</td>
<td>8.1936</td>
<td>-</td>
</tr>
<tr>
<td>Rotor inertia number</td>
<td>$\eta_\beta$</td>
<td>1.0242</td>
<td>-</td>
</tr>
<tr>
<td>Flap frequency ratio</td>
<td>$\lambda_\beta$</td>
<td>1</td>
<td>-</td>
</tr>
<tr>
<td>Linear blade twist</td>
<td>$\Theta_{tw}$</td>
<td>-0.3142</td>
<td>rad</td>
</tr>
<tr>
<td>Blade chord</td>
<td>$c$</td>
<td>0.53</td>
<td>m</td>
</tr>
<tr>
<td>Flapping spring constant</td>
<td>$K_\beta$</td>
<td>0</td>
<td>-</td>
</tr>
<tr>
<td>Main rotor hub stationline</td>
<td>$xh_{sta}$</td>
<td>8.7</td>
<td>m</td>
</tr>
<tr>
<td>Main rotor hub waterline</td>
<td>$zh_{sta}$</td>
<td>8</td>
<td>m</td>
</tr>
<tr>
<td>Stiffness number</td>
<td>$S_\beta$</td>
<td>0</td>
<td>-</td>
</tr>
<tr>
<td>Blade profile drag coefficient</td>
<td>$\delta_0$</td>
<td>-0.0216</td>
<td>-</td>
</tr>
<tr>
<td>Air density</td>
<td>$\rho$</td>
<td>1.2745</td>
<td>kg/m$^3$</td>
</tr>
<tr>
<td>Blade flapping moment of inertia</td>
<td>$I_\beta$</td>
<td>4.2030</td>
<td>kg m$^2$</td>
</tr>
</tbody>
</table>
### Table B-4: Empennage

<table>
<thead>
<tr>
<th>Description</th>
<th>Symbol</th>
<th>UH-60 value</th>
<th>Units</th>
</tr>
</thead>
<tbody>
<tr>
<td>Tail plane area</td>
<td>$S_{TP}$</td>
<td>13.7</td>
<td>$m^2$</td>
</tr>
<tr>
<td>Tail plane stationline</td>
<td>$l_{fn_{sta}}$</td>
<td>17.8</td>
<td>$m$</td>
</tr>
<tr>
<td>Tail plane waterline</td>
<td>$h_{fn_{wat}}$</td>
<td>6.3</td>
<td>$m$</td>
</tr>
<tr>
<td>Fin area</td>
<td>$S_{FN}$</td>
<td>9.8</td>
<td>$m^2$</td>
</tr>
<tr>
<td>Fin stationline</td>
<td>$l_{fp_{sta}}$</td>
<td>17.7</td>
<td>$m$</td>
</tr>
<tr>
<td>Fin waterline</td>
<td>$h_{fp_{wat}}$</td>
<td>6.9</td>
<td>$m$</td>
</tr>
</tbody>
</table>

### Table B-5: Tail rotor

<table>
<thead>
<tr>
<th>Description</th>
<th>Symbol</th>
<th>UH-60 value</th>
<th>Units</th>
</tr>
</thead>
<tbody>
<tr>
<td>Tail rotor speed</td>
<td>$\Omega_T$</td>
<td>124.62</td>
<td>rad/sec</td>
</tr>
<tr>
<td>Tail rotor blade radius</td>
<td>$R_T$</td>
<td>1.67</td>
<td>$m$</td>
</tr>
<tr>
<td>Blade lift curve slope</td>
<td>$\alpha_T$</td>
<td>5.73</td>
<td>rad$^{-1}$</td>
</tr>
<tr>
<td>Tail rotor solidity</td>
<td>$s_T$</td>
<td>0.1875</td>
<td>-</td>
</tr>
<tr>
<td>Blade Lock number</td>
<td>$\gamma_T$</td>
<td>3.378</td>
<td>-</td>
</tr>
<tr>
<td>Tail rotor inertia number</td>
<td>$\eta_{i_T}$</td>
<td>0.4223</td>
<td>-</td>
</tr>
<tr>
<td>Flap frequency ratio</td>
<td>$\lambda_{\beta_T}$</td>
<td>1</td>
<td>-</td>
</tr>
<tr>
<td>Linear blade twist</td>
<td>$\Theta_{tw}$</td>
<td>-0.3142</td>
<td>rad</td>
</tr>
<tr>
<td>Tail rotor hub stationline</td>
<td>$l_{T_{sta}}$</td>
<td>18.6</td>
<td>$m$</td>
</tr>
<tr>
<td>Tail rotor hub waterline</td>
<td>$h_{T_{wat}}$</td>
<td>8.2</td>
<td>$m$</td>
</tr>
<tr>
<td>Blade profile drag coefficient</td>
<td>$\delta_{0T}$</td>
<td>-0.0216</td>
<td>-</td>
</tr>
</tbody>
</table>


